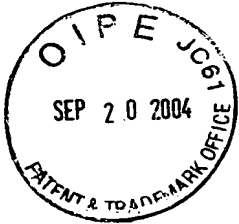


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**In The United States Patent and Trademark Office  
On Appeal From The Examiner To The Board  
of Patent Appeals and Interferences**

In re Application of:      Ranjit N. Notani  
Serial No.:                      09/398,170  
Filing Date:                    September 17, 1999  
Examiner:                        Pedro R. Kanof  
Group Art Unit:                3628  
Title:                                *System and Method for Multi-Enterprise  
Supply Chain Optimization*

**Mail Stop: Appeal Brief**  
Commissioner For Patents  
P.O. Box 1450  
Alexandria, Virginia 22313-1450

Dear Sir:

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**Appeal Brief**

Appellant has appealed to the Board of Patent Appeals and Interferences from the decision of the Examiner mailed May 19, 2004, finally rejecting Claims 1-39. Appellant filed a Notice of Appeal on July 20, 2004. Appellant respectfully submits this Appeal Brief in triplicate with the statutory fee of \$330.00.

DAL01:815398.1

<p style="text-align: center;">CERTIFICATE OF MAILING BY EXPRESS MAIL</p> <p>I hereby certify that this communication is being deposited with the United States Postal Service "Express Mail Post Office to Addressee" under 37 C.F.R. § 1.10 on the date indicated below and is addressed to The Commissioner For Patents, Mail Stop: AF, P.O. Box 1450, Alexandria, Virginia 22313-1450.</p> <p style="text-align: center;"><i>Willie Jiles</i></p> <p>Willie Jiles</p> <p>Date: September 20, 2004</p> <p>Exp. Mail Receipt No. EV 322185335 US</p>
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**Real Party In Interest**

This application is currently owned by i2 Technologies US, Inc., as indicated by:  
an assignment recorded on September 17, 2001, in the Assignment Records of the United States Patent and Trademark Office at Reel 010560, Frames 0066-0068; and  
an assignment recorded on September 30, 2001, in the Assignment Records of the United States Patent and Trademark Office at Reel 012024, Frames 0918-0930.

**Related Appeals and Interferences**

There are no known appeals or interferences which will directly affect or be directly affected by or have a bearing on the Board's decision regarding this appeal.

**Status of Claims**

Claims 1-39 are pending in this application and all stand rejected under a final Office Action mailed May 19, 2004. Appellant presents Claims 1-39 for appeal. Appendix A shows all pending claims.

**Status of Amendments**

The Examiner has entered all amendments submitted before the final Office Action mailed May 19, 2004.

**Summary of Invention**

In particular embodiments, buyer computers 20 collaborate with seller computers 40 to negotiate the supply of products from sellers associated with seller computers 40 to buyers associated with buyer computers 20. (Page 6, Lines 4-7). Buyer computers 20 and seller computers 40 cooperate with each other toward an optimum supply chain between the buyers and the sellers. (Page 6, Lines 7-9). Through an interactive negotiation process and contractual monitoring system, particular embodiments eliminate mismatches between buyer demand and seller supply, which may increase profitability at the buyer and the seller. (Page 6, Lines 9-12).

In particular embodiments, a buyer computer 20 executes a procurement manager 34 that facilitates negotiating and executing supply chain contracts with various seller computers 40. (Page 7, Lines 3-9). Like a buyer computer 20, a seller computer 40 may execute a supply manager 54 that facilitates negotiating and executing supply chain contracts with various buyer computers 20. (Page 7, Lines 20-23). Procurement manager 34 and supply manager 54 may include negotiation modules 114 and 214 that negotiate, between buyer computer 20 and seller computer 40, one or more terms of an option contract. (Page 11, Lines 13-16). Negotiation modules 114 and 214 may be configurable to conduct all or only part of a negotiation between buyer computer 20 and seller computer 40 without receiving any user input. (Page 11, Lines 16-18). Procurement manager 34 and supply manager 54 may include execution modules 119 and 219 that execute negotiated option contracts and store the option contracts. (Page 12, Lines 7-10). Procurement manager 34 and supply manager 54 may include tracking module 120 and 220 that monitor and enforce terms of executed option contracts. (Page 12, Lines 11-14).

Particular embodiments facilitate collaborative negotiation between a buyer computer 20 and a seller computer 40 to generate an electronic option contract for the supply of particular products to a buyer. (Page 9, Lines 3-5). Buyer computer 20 or seller computer 40 may transmit proposed contract terms to the other computer to initiate a negotiation. (Page 9, Lines 5-7). The option contract may include an option specifying buyer obligations with respect to the purchase of a product and seller obligations with respect to the supply of the product. (Page 9, Lines 10-13). In particular embodiments, the option contract specifies a minimum buyer obligation to purchase the product and a maximum seller obligation to supply the product. (Page 9, Lines 13-15). In particular embodiments, the option contract includes an option price payable from the buyer to the seller for the option and includes one or more penalty provisions for violating terms of the option contract. (Page 9, Lines 17-20).

In particular embodiments, after negotiation of the option contract, the option contract is executed and buyer computer 20 and seller computer 40 both store the executed option contract. (Page 10, Lines 3-5). Storing the option contract facilitates monitoring fulfillment of contractual obligations of the buyer and the seller. (Page 10, Lines 5-6). To enforce terms of the option contract, particular embodiments may prohibit buyer computer 20 from exercising an option under the option contract inconsistent with one or more terms of the option contract or may assess a penalty for violating one or more terms of the option contract. (Page 10, Lines 6-9).

In particular embodiments, tracking module 120 at buyer computer 20 may determine whether an option period under an option contract has begun. (Page 20, Lines 15-16). If the option period has begun, tracking module 120 may, automatically or in response to user input, request an update of forecasted demand at the buyer. (Page 20, Lines 16-18). Forecasting module 112 at buyer computer 20 may then update forecasted demand at the buyer according to current demand needs of the buyer. (Page 20, Lines 18-19). Exercise module 121 at buyer computer 20 may then communicate a request to seller computer 40 to exercise an option under the option contract according to the updated forecasted demand. (Page 20, Lines 19-21). Tracking module 120 may then determine whether the exercise of the option satisfies a minimum obligation of the buyer under the option contract. (Page 20, Lines 23-25). If the buyer has not met the minimum obligation under the option contract, tracking module 120 may determine whether an exercise period under the option contract has ended. (Page 20, Lines 25-26). If the exercise period has ended and the buyer has not met the minimum obligation under the option contract, tracking module 120 may determine a penalty resulting from the buyer not meeting the minimum obligation. (Page 20, Lines 26-28). On the other hand, if tracking module 120 determines that the exercise period has not yet ended, tracking module 120 may wait for another update to forecasted demand at the buyer. (Page 20, Lines 29-31).

If the buyer has met the minimum obligation, the option contract may allow buyer computer 20 to place further orders before the exercise period expires. (Page 21, Lines 1-3). As an example, procurement manager 34 at buyer computer 20 may determine that, even though the buyer has met the minimum obligation, the exercise period has not yet ended. (Page 21, Lines 3-5). If buyer computer 20 receives input from a user or input from a memory unit indicating that the buyer should procure additional product, procurement manager 34 may communicate an update of forecasted demand at the buyer requesting additional product. (Page 21, Lines 5-8). In particular embodiments, procurement manager 34 may decline to communicate the updated forecast of demand at the buyer if procurement manager 34 determines that the seller has already met the maximum obligation of the seller under the option contract. (Page 21, Lines 8-10).

In particular embodiments, supply manager 54 at seller computer 40 may receive a request from buyer computer 20 to exercise an option under the option contract. (Page 22, Lines 5-8). Supply manager 54 may then access stored terms of the option contract to determine whether an exercise period under the option contract has begun. (Page 22, Lines 5-9). If supply manager 54 determines that the exercise period has not yet begun, supply manager 54 notifies buyer computer 20 that the request is improper, in this case because the request is premature. (Page 22, Lines 9-11). If supply manager 54 determines that the exercise period has begun, supply manager 54 may determine whether the request exceeds a maximum obligation of the seller under the option contract. (Page 22, Lines 13-14). If supply manager 54 determines that the request from buyer computer 20 exceeds the maximum obligation of the seller, supply manager 54 may notify buyer computer 20 that the request is improper. (Page 22, Lines 18-21). If supply manager 54 determines that the request does not exceed the maximum obligation of the seller, supply manager 54 may store the request. (Page 22, Lines 25-27). Supply manager 54 may then determine whether the exercise period under the option contract has ended. (Page 22, Lines 27-28). If supply manager 54 determines that the exercise period has not yet ended, supply manager 54 may

receive another request from buyer computer 20 and process the request as described above. (Page 22, Line 30, through Page 23, Line 2).

If supply manager 54 determines that the exercise period has already ended, supply manager 54 may determine whether the buyer has met a minimum obligation of the buyer under the option contract. (Page 23, Lines 3-4). If supply manager 54 determines that the buyer has not met the minimum obligation, supply manager 54 may determine a penalty for the buyer not meeting the minimum obligation according to a penalty provision in the option contract. (Page 23, Lines 6-9). Tracking module 220 at seller computer 40 may access terms of the option contract stored at seller computer 40 to identify the penalty provision, compare the penalty provision with the short fall of the buyer, and calculate a resulting penalty. (Page 23, Lines 9-11).

#### **Statement of Issue**

Are Claims 1-39 patentable over *Purchasing Handbook* by George W. Aljian (“*Aljian*”) in view of *Contingent Claims Contracting for Purchasing Decisions in Inventory Management* by Peter H. Ritchken and Charles S. Tapiero (“*Ritchken*”) and in further view of U.S. Patent No. 6,247,774 to Roden et al. (“*Roden*”) under 35 U.S.C. § 103(a)?

#### **Grouping of Claims**

Appellant has made an effort to group claims together to reduce the burden on the Board. Appellant has concluded that all claims do not stand or fall together. In the argument section of this Brief, Appellant presents reasons why the claims in each group are separately patentable from the claims in the other group. Appellant has concluded that the Board may group the claims as follows:

1. Group 1 may include Claims 1-11, 19-27, and 38-39; and
2. Group 2 may include Claims 12-18 and 18-37.

**Argument**

The rejection of Claims 1-39 under *Aljian* in view of *Ritchken* and in further view of *Roden* is improper, and the Board should withdraw the rejection.

**Claims 1-39 are Allowable over the Proposed  
*Aljian-Ritchken-Roden* Combination**

***A. Overview***

The Examiner rejects Claims 1-39 under *Aljian* in view of *Ritchken* and in further view of *Roden*. Appendix B includes a copy of *Aljian*, Appendix C includes a copy of *Ritchken*, and Appendix D includes a copy of *Roden*. Appellant respectfully submits that Claims 1-39 are clearly allowable over the proposed *Aljian-Ritchken-Roden* combination.

***B. Standard***

The question raised under 35 U.S.C. § 103 is whether the prior art taken as a whole would suggest the claimed invention taken as a whole to one of ordinary skill in the art at the time of the invention. *See* 35 U.S.C. § 103(a) (2000). Accordingly, even if all elements of a claim are disclosed in various prior art references, which is certainly not the case here as discussed below, the claimed invention taken as a whole cannot be said to be obvious without some reason given in the prior art why one of ordinary skill at the time of the invention would have been prompted to modify the teachings of a reference or combine the teachings of multiple references to arrive at the claimed invention.

The M.P.E.P. sets forth the strict legal standard for establishing a *prima facie* case of obviousness based on modification or combination of prior art references:

To establish a *prima facie* case of obviousness, three basic criteria must be met. First, there must be some suggestion or motivation, either in the references themselves or in the knowledge generally available to one of ordinary skill in the art, to modify the reference or combine reference teachings. Second, there must be a reasonable expectation of success. Finally, the prior art reference (or references where combined) must teach or suggest all the claim limitations.

M.P.E.P. chs. 2142-43 (Rev. 2, May 2004). “To establish *prima facie* obviousness of a claimed invention, all the claim limitations must be taught or suggested by the prior art. All words in a claim must be considered in judging the patentability of that claim against the prior art.” M.P.E.P. ch. 2143.03 (Rev. 2, May 2004) (citations omitted).

In addition, the M.P.E.P. and the Federal Circuit repeatedly warn against using an applicant’s disclosure as a blueprint to reconstruct the claimed invention. For example, the M.P.E.P. states, “The tendency to resort to ‘hindsight’ based upon applicant’s disclosure is often difficult to avoid due to the very nature of the examination process. However, impermissible hindsight must be avoided and the legal conclusion must be reached on the basis of the facts gleaned from the prior art.” M.P.E.P. ch. 2142 (Rev. 2, May 2004). The governing Federal Circuit cases are equally clear.

A critical step in analyzing the patentability of claims pursuant to [35 U.S.C. § 103] is casting the mind back to the time of invention, to consider the thinking of one of ordinary skill in the art, guided only by the prior art references and the then-accepted wisdom in the field. . . . Close adherence to this methodology is especially important in cases where the very ease with which the invention can be understood may prompt one “to fall victim to the insidious effect of a hindsight syndrome wherein that which only the invention taught is used against its teacher.”

*In re Kotzab*, 217 F.3d 1365, 1369, 55 U.S.PQ.2d 1313, 1316 (Fed. Cir. 2000) (citations omitted).



**C. *Aljian***

*Aljian* merely discloses factors (such as requirements, quantity in stock and on order, procurement time, obsolescence, ordering cost, and inventory carrying cost) influencing how much inventory to procure and when to procure inventory. (Pages 13-5 through 13-33). *Aljian* also discloses a mechanical order-writing system and various types of contracts, such as indefinite-quantity, or open-end, contracts. (Pages 14-28 through 14-36 and 19-52 through 19-54).

**D. *Ritchken***

*Ritchken* merely discloses designing contingent claims (options) including purchasing commitments to meet certain risk-reward preferences. (Pages 864-70).

**E. *Roden***

*Roden* merely discloses a supplier modifying a forecast of demand according to a quantity of items that a company (other than the supplier) has dispensed to consumers and a frequency at which the company dispensed the items. (Abstract; Column 10, Lines 1-7; Column 11, Lines 1-16).

**F. *Group 1 (Claims 1-11, 19-27, and 38-39)***

The Examiner rejects Claims 1-11, 19-27, and 38-39 under *Aljian* in view of *Ritchken* and in further view of *Roden*. Appellant respectfully submits that Claims 1-11, 19-27, and 38-39 are allowable over the proposed *Aljian-Ritchken-Roden* combination.

Claims 1-11, 19-27, and 38-39 are separately patentable from every other claim subject to the same rejection and recite limitations that are substantially different from limitations recited in all other claims. Therefore, Appellant has grouped Claims 1-11, 19-27, and 38-39 together separately from all other claims.

Independent Claim 1 recites:

A method of optimizing multi-enterprise supply chain agreements using an electronic option contract, the method comprising:

determining at a buyer computer a range of forecasted demand for a product;

communicating from the buyer computer to a seller computer an offer to enter into an option contract for the supply of a product, the option contract including an option corresponding to the range of forecasted demand;

executing the option contract;

receiving at the buyer computer an indication of current buyer demand for the product;

automatically and without user input subsequent to receiving at the buyer computer the indication of current buyer demand for the product, determining at the buyer computer whether the indicated current buyer demand exceeds a maximum option quantity specified in the option contract; and

automatically and without user input subsequent to determining at the buyer computer whether the indicated current buyer demand exceeds the maximum option quantity specified in the option contract, if the indicated current buyer demand does not exceed the maximum option quantity specified in the option contract, communicating from the buyer computer to the seller computer a request to exercise at least a portion of the option based at least in part on the indicated current buyer demand.

Independent Claim 19 recites limitations substantially similar to limitations recited in independent Claim 1.

Even assuming for the sake of argument that *Aljian*, *Ritchken*, and *Roden* could be combined with each other as the Examiner proposes, the proposed *Aljian-Ritchken-Roden* combination would still fail to disclose, teach, or suggest limitations recited in independent Claim 1.

The Examiner acknowledges that *Aljian* fails to disclose, teach, or suggest, the following limitations recited in independent Claim 1:

- ***receiving at the buyer computer an indication of current buyer demand***

*for the product;*

- *automatically and without user input subsequent to receiving at the buyer computer the indication of current buyer demand for the product, determining at the buyer computer whether the indicated current buyer demand exceeds a maximum option quantity specified in the option contract; and*
- *automatically and without user input subsequent to determining at the buyer computer whether the indicated current buyer demand exceeds the maximum option quantity specified in the option contract, if the indicated current buyer demand does not exceed the maximum option quantity specified in the option contract, communicating from the buyer computer to the seller computer a request to exercise at least a portion of the option based at least in part on the indicated current buyer demand.*

To make up for the fact that *Aljian* fails to disclose, teach, or suggest the first of these limitations, the Examiner asserts that *Roden* discloses *receiving at the buyer computer an indication of current buyer demand for the product*, as recited in independent Claim 1. *Roden* clearly fails to disclose, teach, or suggest this limitation. Instead, as Applicants have pointed out, *Roden* merely discloses a supplier modifying, after an initial period of time, a forecast of demand according to a quantity of items that a company (other than the supplier) has dispensed to consumers and a frequency at which the company dispensed the items.

To make up for the fact that *Aljian* fails to disclose, teach, or suggest the other limitations indicated above, the Examiner takes Official Notice that these limitations “are old and well known within the management of any type of contract, and in the options art in particular. Therefore, it would have been obvious to one having ordinary skill in the art to include those steps.” To support the Examiner’s conclusion that these limitations were old and well known at the time of invention, the Examiner cites *Modeling Take-or-Pay Contract Decisions* by Carl R. Shultz (“*Shultz*”). Appellant notes that *Shultz* merely discloses that a take-or-pay contract is a mixture of a requirements contract and an indefinite quantity contract obligating a buyer to purchase between high and low quantity limits and that, under a take-or-pay contract, a buyer incurs a penalty if purchases from the buyer fall short of the low quantity limit. (Pages 2-3).

The Examiner concludes that, because *Schultz* discloses such high and low quantity limits, *Schultz* also discloses “checking whether a buyer demand exceeds a maximum option quantity or falls within agreed low and high limits.” Appellant respectfully disagrees with the Examiner. Nowhere does *Shultz* disclose, teach, or suggest any such “checking,” much less ***determining whether the indicated current buyer demand exceeds a maximum option quantity specified in the option contract***, as recited in independent Claim 1. Appellant notes that the Examiner neither explains how “checking whether a buyer demand exceeds a maximum option quantity falls within agreed low and high limits” logically follows from high and low quantity limits in take-or-pay contracts in *Schultz* nor points out exactly where *Schultz* allegedly provides any disclosure, teaching, or suggestion of such checking. Even assuming for the sake of argument that *Schultz* disclosed such checking and that such checking could be properly considered ***determining whether the indicated current buyer demand exceeds a maximum option quantity specified in the option contract***, as recited in independent Claim 1, *Schultz* would still fail to disclose, teach, or suggest ***determining at the buyer computer whether the indicated current buyer demand exceeds a maximum option quantity specified in the option contract***.

After concluding that *Schultz* discloses “checking whether a buyer demand exceeds a maximum option quantity falls within agreed low and high limits,” the Examiner then asserts that the following limitation, as recited in independent Claim 1, “would have been obvious to one of ordinary skill in the art”:

- ***subsequent to determining whether the indicated current buyer demand exceeds the maximum option quantity specified in the option contract, if the indicated current buyer demand does not exceed the maximum option quantity specified in the option contract, communicating a request to exercise at least a portion of the option based at least in part on the indicated current buyer demand***

Appellant respectfully disagrees with the Examiner. Contrary to the requirements of governing Federal Circuit case law and the M.P.E.P., the Examiner provides no rationale or

evidence whatsoever supporting the Examiner's assertion that the above limitation, as recited in independent Claim 1, "would have been obvious to one of ordinary skill in the art." According to the M.P.E.P., "Ordinarily, there must be some form of evidence in the record to support an assertion of common knowledge." M.P.E.P. ch. 2143.03(B) (Rev. 2, May 2004). Moreover, "It is never appropriate to rely solely on "common knowledge" in the art without evidentiary support in the record, as the principal evidence upon which a rejection was based." *Id.* at ch. 2143.03(A). "The examiner must provide specific factual findings predicated on sound technical and scientific reasoning to support his or her conclusion of common knowledge." *Id.* at ch. 2143.03(B). If the Examiner intends to rely on information that was generally available to a person having ordinary skill in the art at the time of the invention to demonstrate that the above limitation, as recited in independent Claim 1, would have been obvious to a person having ordinary skill in the art at the time of the invention, the Examiner must provide documentary evidence that such information was in fact generally available to a person having ordinary skill in the art at the time of the invention, as governing Federal Circuit case law and the M.P.E.P. require. The Examiner has failed to do so.

Next, the Examiner asserts, "One of ordinary skill in the art would have turned to Roden et al to use a computer to perform these steps as Roden et al is directed to a contract system between a buyer and a distributor of goods/services." Appellant respectfully disagrees with the Examiner. The Examiner fails to point out exactly where *Roden* allegedly discloses, teaches, or suggests a "contract system between a buyer and a distributor of goods/services," and Appellant submits that *Roden* in fact makes no such disclosure, teaching, or suggestion. As discussed above, *Roden* merely discloses a supplier modifying a forecast of demand according to a quantity of items that a company (other than the supplier) has dispensed to consumers and a frequency at which the company dispensed the items. Even assuming for the sake of argument that *Roden* disclosed a "contract system between a buyer and a distributor of goods/services," *Roden* would still fail to disclose, teach, or even suggest, as recited in independent Claim 1:

- ***determining at the buyer computer whether the indicated current buyer***

*demand exceeds a maximum option quantity specified in the option contract; and*

- *if the indicated current buyer demand does not exceed the maximum option quantity specified in the option contract, communicating from the buyer computer to the seller computer a request to exercise at least a portion of the option based at least in part on the indicated current buyer demand.*

Appellant notes that the Examiner fails to explain how “one of ordinary skill in the art” would have relied on *Roden* as the Examiner proposes. With respect to combining *Roden* with *Schultz* and what the Examiner asserts without any supporting rationale or supporting evidence “would have been obvious to one of ordinary skill in the art,” the Examiner merely states, “One of ordinary skill in the art would have turned to Roden et al . . . as Roden et al is directed to a contract system between a buyer and a distributor of goods/services.” Appellant respectfully submits that, for at least the following reasons, such an assertion fails to demonstrate that *Roden*, *Shultz*, what the Examiner asserts “would have been obvious to one of ordinary skill in the art,” or knowledge generally available to a person having ordinary skill in the art at the time of the invention provide any teaching, suggestion, or motivation to make the proposed combination, as governing Federal Circuit case law and the M.P.E.P. require. Appellant further respectfully submits that such an assertion fails to demonstrate that a person having ordinary skill in the art at the time of the invention would have reasonably expected the proposed combination to achieve the purported results, as governing Federal Circuit case law and the M.P.E.P. further require.

Nowhere does the Examiner demonstrate, with respect to the proposed combination, that *Roden*, *Schultz*, what the Examiner asserts “would have been obvious to one of ordinary skill in the art,” or knowledge generally available to a person having ordinary skill in the art at the time of the invention provide any teaching, suggestion, or motivation whatsoever to make the proposed combination. The Examiner merely asserts, “One of ordinary skill in the art would have turned to Roden et al . . . as Roden et al is directed to a contract system between a buyer and a distributor of goods/services.” The Examiner does not even attempt to

demonstrate that such a teaching, suggestion, or motivation can be found in *Roden, Schultz*, what the Examiner asserts “would have been obvious to one of ordinary skill in the art,” or knowledge generally available to a person having ordinary skill in the art at the time of the invention. If the Examiner intends to assert that a teaching, suggestion, or motivation to rely on *Roden* as the Examiner proposes could have been found in information generally available to a person having ordinary skill in the art at the time of the invention, the Examiner must provide documentary evidence that such information was in fact generally available to a person having ordinary skill in the art at the time of the invention, as governing Federal Circuit case law and the M.P.E.P. require. The Examiner has failed to do so.

Moreover, nowhere does the Examiner demonstrate that a person having ordinary skill in the art at the time of the invention would have reasonably expected the proposed combination to achieve the purported results. First, the Examiner fails to demonstrate that the proposed combination would have in fact achieved the purported results. Nowhere does the Examiner even attempt to demonstrate that combining *Roden* with *Schultz* and what the Examiner asserts “would have been obvious to one of ordinary skill in the art” would have actually enabled, as recited in independent Claim 1:

- *determining at the buyer computer whether the indicated current buyer demand exceeds a maximum option quantity specified in the option contract; and*
- *if the indicated current buyer demand does not exceed the maximum option quantity specified in the option contract, communicating from the buyer computer to the seller computer a request to exercise at least a portion of the option based at least in part on the indicated current buyer demand.*

Second, even assuming for the sake of argument that the proposed combination would have produced the purported results, the Examiner fails to demonstrate that a person having ordinary skill in the art at the time of the invention would have reasonably expected such results. The Examiner merely asserts, “One of ordinary skill in the art would have turned to *Roden et al* to use a computer to perform these steps.” The Examiner does not even attempt

to demonstrate that a person having ordinary skill in the art at the time of the invention would have reasonably expected the system disclosed in *Roden*, when combined with *Schultz* and what the Examiner asserts “would have been obvious to one of ordinary skill in the art,” to enable “these steps.” If the Examiner intends to rely on information that was generally available to a person having ordinary skill in the art at the time of the invention to demonstrate that a person having ordinary skill in the art at the time of the invention would have expected these purported results, the Examiner must provide documentary evidence that such information was in fact generally available to a person having ordinary skill in the art at the time of the invention, as governing Federal Circuit case law and the M.P.E.P. require. The Examiner has failed to do so.

After asserting that “[o]ne of ordinary skill in the art would have turned to *Roden et al* to use a computer to perform these steps,” the Examiner further asserts:

The “automatically and without user input” steps or functions would have been obvious to one of ordinary skill in the art to do when programming the combined systems noted above. The “automatically and without user input” functions would have expedited an acknowledgement to perform a buyer or seller request.

Appellant respectfully disagrees with the Examiner. The Examiner merely asserts that, because the limitation indicated above would have, according to the Examiner, provided an advantage, i.e., “expedit[ing] an acknowledgement to perform a buyer or seller request,” the limitation indicated above would have been obvious to a person having ordinary skill in the art at the time of the invention. Appellant respectfully submits that such rationale is entirely insufficient under governing Federal Circuit case law and the M.P.E.P. to support an obviousness rejection. Moreover, contrary to the requirements of governing Federal Circuit case law and the M.P.E.P., the Examiner provides no evidence whatsoever supporting the Examiner’s assertion that the limitation indicated above “would have been obvious to one of ordinary skill in the art.” Appellant reiterates that, according to the M.P.E.P., “Ordinarily, there must be some form of evidence in the record to support an assertion of common knowledge.” M.P.E.P. ch. 2143.03(B) (Rev. 2, May 2004). Moreover, “It is never



appropriate to rely solely on “common knowledge” in the art without evidentiary support in the record, as the principal evidence upon which a rejection was based.” *Id.* at ch. 2143.03(A). “The examiner must provide specific factual findings predicated on sound technical and scientific reasoning to support his or her conclusion of common knowledge.” *Id.* at ch. 2143.03(B). If the Examiner intends to rely on information that was generally available to a person having ordinary skill in the art at the time of the invention to demonstrate that the limitation indicated above would have been obvious to a person having ordinary skill in the art at the time of the invention, the Examiner must provide documentary evidence that such information was in fact generally available to a person having ordinary skill in the art at the time of the invention, as governing Federal Circuit case law and the M.P.E.P. require. The Examiner has failed to do so.

For at least these reasons, *Schultz* fails to support the Examiner’s conclusion that the other limitations above were old and well known at the time of invention. Appellant therefore reiterates that the Examiner cannot properly take Official Notice that the other limitations above “are old and well known within the management of any type of contract, and in the options art in particular. Therefore, it would have been obvious to one having ordinary skill in the art to include those steps.” As Appellant pointed out in Appellant’s January 21, 2004, Response, according to the M.P.E.P., “the notice of facts beyond the record which may be taken by the examiner must be ‘capable of such instant and unquestionable demonstration as to defy dispute.’” M.P.E.P. ch. 2144.03(A) (Rev. 1, Feb. 2003). The M.P.E.P. continues:

It would not be appropriate for the examiner to take official notice of facts without citing a prior art reference where the facts asserted to be well known are not capable of instant and unquestionable demonstration as being well-known. For example, assertions of technical facts in the areas of esoteric technology or specific knowledge of the prior art must always be supported by citation to some reference work recognized as standard in the pertinent art.

*Id.* (emphasis in original). Furthermore, “[i]f such notice is taken, the basis for such reasoning must be set forth explicitly. The examiner must provide specific factual findings predicated on sound technical and scientific reasoning to support his or her conclusion of common knowledge.” M.P.E.P. ch. 2144.03(B) (Rev. 1, Feb. 2003). The M.P.E.P. also

states that, “[i]f applicant adequately traverses the examiner’s assertion of official notice, the examiner must provide documentary evidence in the next Office action if the rejection is to be maintained.” M.P.E.P. ch. 2144.03(C) (Rev. 1, Feb. 2003).

For at least these reasons, the proposed *Aljian-Ritchken-Roden* combination fails to disclose, teach, or suggest all elements of independent Claims 1 and 19. Independent Claims 1 and 19 are therefore allowable over the proposed *Aljian-Ritchken-Roden* combination. Because dependent Claims 2-11 and 38 depend on independent Claim 1 and dependent Claims 20-27 and 39 depend on independent Claim 19, dependent Claims 2-11, 20-27, and 38-39 are therefore also allowable over the proposed *Aljian-Ritchken-Roden* combination.

**G. Group 2 (Claims 12-18 and 18-37)**

The Examiner rejects Claims 12-18 and 18-37 under *Aljian* in view of *Ritchken* and in further view of *Roden*. Appellants respectfully submit that Claims 12-18 and 18-37 are allowable over the proposed *Aljian-Ritchken-Roden* combination.

Claims 12-18 and 18-37 are separately patentable from every other claim subject to the same rejection and recite limitations that are substantially different from limitations recited in all other claims. Therefore, Appellants have grouped Claims 12-18 and 18-37 together separately from all other claims.

Independent Claim 12 recites:

A method of optimizing multi-enterprise supply chain agreements using an electronic option contract, the method comprising:

receiving at a seller computer terms of an option contract from a buyer computer, the terms comprising an option corresponding to a buyer’s range of forecasted demand for a product;

communicating to the buyer computer an acceptance of the terms of the option contract;

storing the terms of the accepted option contract in a memory accessible to the seller computer;

receiving, at the seller computer and from the buyer computer, a request to exercise at least a portion of the option based at least in part on an indication of current buyer demand for the product;

at the seller computer, automatically and without user input subsequent to receiving at the seller computer the request to exercise at least the portion of the option, in response to receiving the request:

accessing the stored terms of the option contract; and

using the stored terms of the option contract:

determining whether an option period specified in the option contract has begun;

if the option period has not yet begun, notifying the buyer computer that the request is premature; and

if the option period has begun:

determining whether the request specifies a request quantity that exceeds a maximum option quantity specified in the option contract;

if the request quantity exceeds the maximum option quantity, notifying the buyer computer that the request is improper; and

if the request quantity does not exceed the maximum option quantity, storing the request for seller compliance.

Independent Claim 28 recites limitations substantially similar to limitations recited in independent Claim 12.

Even assuming for the sake of argument that *Aljian*, *Ritchken*, and *Roden* could be combined with each other as the Examiner proposes, the proposed *Aljian-Ritchken-Roden* combination would still fail to disclose, teach, or suggest limitations recited in independent Claim 12.

The Examiner acknowledges that *Aljian* fails to disclose, teach, or suggest, the following limitations recited in independent Claim 12:

- ***receiving, at the seller computer and from the buyer computer, a request to exercise at least a portion of the option based at least in part on an indication of current buyer demand for the product;***

- *at the seller computer, automatically and without user input subsequent to receiving at the seller computer the request to exercise at least the portion of the option, in response to receiving the request:*
  - *accessing the stored terms of the option contract; and*
  - *using the stored terms of the option contract:*
    - *determining whether an option period specified in the option contract has begun;*
    - *if the option period has not yet begun, notifying the buyer computer that the request is premature; and*
    - *if the option period has begun:*
      - *determining whether the request specifies a request quantity that exceeds a maximum option quantity specified in the option contract;*
      - *if the request quantity exceeds the maximum option quantity, notifying the buyer computer that the request is improper; and*
      - *if the request quantity does not exceed the maximum option quantity, storing the request for seller compliance.*

To make up for the fact that *Aljian* fails to disclose, teach, or suggest the first of these limitations, the Examiner asserts that *Roden* discloses *receiving, at the seller computer and from the buyer computer, a request to exercise at least a portion of the option based at least in part on an indication of current buyer demand for the product*, as recited in independent Claim 12. *Roden* clearly fails to disclose, teach, or suggest this limitation. Instead, as Applicants have pointed out, *Roden* merely discloses a supplier modifying, after an initial period of time, a forecast of demand according to a quantity of items that a company (other than the supplier) has dispensed to consumers and a frequency at which the company dispensed the items.

To make up for the fact that *Aljian* fails to disclose, teach, or suggest the other limitations indicated above, the Examiner takes Official Notice that these limitations “are old and well known within the management of any type of contract, and in the options art in particular. Therefore, it would have been obvious to one having ordinary skill in the art to

include those steps.” In Appellant’s Response mailed January 21, 2004, Appellant argued that the Examiner could not properly take Official Notice that the other limitations indicated above were “old and well known” at the time of the invention. Appellant notes that, in the Office Action mailed May 19, 2004, the Examiner failed to respond to Appellant’s argument. Instead, the Examiner merely responded to Appellant’s argument in Appellant’s response mailed January 21, 2004, that the Examiner could not properly take Official Notice that particular limitations recited in independent Claim 1 were “old and well known.” The Examiner cites *Schultz* to support the Examiner’s conclusion that particular limitations recited in independent Claim 1 were “old and well known,” as discussed above, but does not indicate in any way how *Schultz* allegedly supports the Examiner’s conclusion that the other limitations indicated above were “old and well known.”

Therefore, Appellant reiterates that the Examiner has asserted that these limitations were old and well known at the time of invention without “provid[ing] specific factual findings predicated on sound technical and scientific reasoning to support [the Examiner’s] conclusion of common knowledge,” as the M.P.E.P. requires. In addition, the Examiner’s assertion that the above limitations were old and well known at the time of invention is hardly “capable of such instant and unquestionable demonstration as to defy dispute,” as the M.P.E.P. further requires. Accordingly, Applicant respectfully submits that the Examiner has improperly taken Official Notice that the above limitations were old and well known at the time of invention. If the Examiner intends to maintain this rejection, the Examiner must provide documentary evidence that the above limitations were old and well known at the time of invention, as the Examiner asserts. The Examiner has failed to do so.

For at least these reasons, the proposed *Aljian-Ritchken-Roden* combination fails to disclose, teach, or suggest all elements of independent Claims 12 and 28. Independent Claims 12 and 28 are therefore allowable over the proposed *Aljian-Ritchken-Roden* combination. Because dependent Claims 13-18 and 29-37 depend on independent Claims 12

and 28, respectively, dependent Claims 13-18 and 29-37 are therefore also allowable over the proposed *Aljian-Ritchken-Roden* combination

**Conclusion**

Appellant has demonstrated that the present invention, as claimed, is clearly distinguishable over the prior art cited by the Examiner. Therefore, Appellant respectfully requests the Board of Patent Appeals and Interferences to reverse the final rejection of the Examiner and instruct the Examiner to issue a notice of allowance of all claims.

Appellant has enclosed a check in the amount of \$330.00 for this Appeal Brief. Appellant believes no additional fees are due. The Commissioner is hereby authorized to charge any fee and credit any overpayment to Deposit Account No. 02-0384 of Baker Botts L.L.P.

Respectfully submitted,

BAKER BOTTS L.L.P.  
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**Appendix A**

1. A method of optimizing multi-enterprise supply chain agreements using an electronic option contract, the method comprising:

determining at a buyer computer a range of forecasted demand for a product;

communicating from the buyer computer to a seller computer an offer to enter into an option contract for the supply of a product, the option contract including an option corresponding to the range of forecasted demand;

executing the option contract;

receiving at the buyer computer an indication of current buyer demand for the product;

automatically and without user input subsequent to receiving at the buyer computer the indication of current buyer demand for the product, determining at the buyer computer whether the indicated current buyer demand exceeds a maximum option quantity specified in the option contract; and

automatically and without user input subsequent to determining at the buyer computer whether the indicated current buyer demand exceeds the maximum option quantity specified in the option contract, if the indicated current buyer demand does not exceed the maximum option quantity specified in the option contract, communicating from the buyer computer to the seller computer a request to exercise at least a portion of the option based at least in part on the indicated current buyer demand.

2. The method of Claim 1, wherein the option comprises a range of parameters selected from a group consisting of:

a minimum quantity of a product that the buyer is obligated to purchase, and a maximum quantity of the product that the seller is obligated to supply;

a minimum number of product types that the buyer is obligated to purchase, and a maximum number of product types that the seller is obligated to supply; and

a minimum number and a maximum number of locations where a product must be delivered.



3. The method of Claim 1, wherein the option comprises a plurality of ranges of parameters each selected from a group consisting of:

a minimum quantity of a product that the buyer is obligated to purchase, and a maximum quantity of the product that the seller is obligated to supply;

a minimum number of product types that the buyer is obligated to purchase, and a maximum number of product types that the seller is obligated to supply; and

a minimum number and a maximum number of locations where a product must be delivered.

4. The method of Claim 1, wherein the option contract includes an exercise period comprising a period of time after the execution of the option contract during which the buyer must exercise its option.

5. The method of Claim 4, wherein exercising the option comprises:  
specifying a first quantity of product desired at a first time during the exercise period;  
specifying a second quantity of product desired at a second time during the exercise period;

and wherein the updated forecasted demand comprises the sum of the first and second quantities of product desired.

6. The method of Claim 1, further comprising:  
receiving from the seller computer a modified range of forecasted demand comprising the range of forecasted demand modified by an optimization model at the seller computer;  
and

accepting the modified range of forecasted demand as a term to the option contract.

7. The method of Claim 1, further comprising:  
receiving a proposed contract term from the seller computer;  
accessing a memory comprising a range of acceptable contract terms; and  
comparing the proposed contract term to the range of acceptable contract terms.

8. The method of Claim 7, further comprising:  
determining that the proposed contract term is within the range of acceptable contract terms; and  
accepting the proposed contract term without user input.

9. The method of Claim 7, further comprising:  
determining that the proposed contract term is not within the range of acceptable contract terms; and  
identifying the proposed contract term as a term requiring user input prior to acceptance.

10. The method of Claim 1, further comprising:  
determining at the buyer computer a proposed option price comprising a value of the option to a buyer associated with the buyer computer;  
communicating from the buyer computer to the seller computer the proposed option price; and  
negotiating with the seller computer an agreed option price based on the value of the option to the buyer and a cost of the option to a seller associated with the seller's computer.

11. The method of Claim 10, wherein negotiating an agreed option price comprises:

receiving from the seller computer a modified proposed range of forecasted demand comprising the proposed range of forecasted demand modified by an optimization model at the seller computer;

determining a modified proposed option price based on the modified proposed range of forecasted demand; and

communicating the modified proposed option price to the seller computer.

12. A method of optimizing multi-enterprise supply chain agreements using an electronic option contract, the method comprising:

receiving at a seller computer terms of an option contract from a buyer computer, the terms comprising an option corresponding to a buyer's range of forecasted demand for a product;

communicating to the buyer computer an acceptance of the terms of the option contract;

storing the terms of the accepted option contract in a memory accessible to the seller computer;

receiving, at the seller computer and from the buyer computer, a request to exercise at least a portion of the option based at least in part on an indication of current buyer demand for the product;

at the seller computer, automatically and without user input subsequent to receiving at the seller computer the request to exercise at least the portion of the option, in response to receiving the request:

accessing the stored terms of the option contract; and

using the stored terms of the option contract:

determining whether an option period specified in the option contract has begun;

if the option period has not yet begun, notifying the buyer computer that the request is premature; and

if the option period has begun:

determining whether the request specifies a request quantity that exceeds a maximum option quantity specified in the option contract;

if the request quantity exceeds the maximum option quantity, notifying the buyer computer that the request is improper; and

if the request quantity does not exceed the maximum option quantity, storing the request for seller compliance.

13. The method of Claim 12, wherein the option comprises a range of parameters selected from a group consisting of:

a minimum quantity of a product that the buyer is obligated to purchase, and a maximum quantity of the product that the seller is obligated to supply;

a minimum number of product types that the buyer is obligated to purchase, and a maximum number of product types that the seller is obligated to supply; and

a minimum number and a maximum number of locations where a product must be delivered.

14. The method of Claim 12, wherein the option comprises a plurality of ranges of parameters each selected from a group consisting of:

a minimum quantity of a product that the buyer is obligated to purchase, and a maximum quantity of the product that the seller is obligated to supply;

a minimum number of product types that the buyer is obligated to purchase, and a maximum number of product types that the seller is obligated to supply; and

a minimum number and a maximum number of locations where a product must be delivered.

15. The method of Claim 12, wherein the option contract includes an exercise period comprising a period of time after the execution of the option contract during which the buyer must exercise its option, and wherein enforcing the terms of the option contract comprises:

receiving a request from the buyer computer to exercise the buyer's option comprising an identification of the buyer's exercised level of demand under the contract;

accessing the memory to retrieve the stored contract terms, including an exercise period begin date and an exercise period end date; and

comparing a current date to the exercise period begin date and the exercise period end date.

16. The method of Claim 15, further comprising:  
determining that the exercise period has begun and has not expired; and  
accepting the buyer computer's request to exercise the buyer's option.

17. The method of Claim 16, wherein the buyer computer's request comprises an identification of a first quantity of product desired, and further comprising:  
storing the request for a first quantity of product desired in the memory;  
receiving a subsequent request from the buyer computer to exercise the buyer's option comprising an identification of a second quantity of product desired;  
determining that the exercise period has not yet expired; and  
storing the request for a second quantity of product desired in the memory.

18. The method of claim 16, further comprising:  
comparing the buyer's exercised demand level to a minimum obligation of the buyer under the contract; and  
determining a penalty if the buyer's minimum obligation level exceeds the buyer's exercised demand level after the expiration of the exercise period.

19. A procurement manager operable to be executed on the processor of a buyer computer, the procurement manager comprising:

- a forecast module operable to determine the buyer's range of forecasted demand for a product;

- a negotiation module operable to communicate to a seller computer an offer to enter into an option contract for the supply of a product, the option contract including a proposed option corresponding to the range of forecasted demand, the negotiation module further operable to receive from the seller computer a modified range of forecasted demand, to communicate the modified range of forecasted demand to the forecast module, and to receive from the forecast module a compromised range of forecasted demand;

- an execution module operable to execute an option contract including an option corresponding to the compromised range of forecasted demand; and

- an exercise module operable to:

- receive from the forecast module an indication of current buyer demand for the product;

- determine whether the indicated current buyer demand exceeds a maximum option quantity specified in the option contract; and

- if the indicated current buyer demand does not exceed the maximum option quantity specified in the option contract, communicate to the seller computer a request to exercise at least a portion of the option based at least in part on the indicated current buyer demand.

20. The procurement manager of Claim 19, wherein the option comprises a range of parameters selected from a group consisting of:

a minimum quantity of a product that the buyer is obligated to purchase, and a maximum quantity of the product that the seller is obligated to supply;

a minimum number of product types that the buyer is obligated to purchase, and a maximum number of product types that the seller is obligated to supply; and

a minimum number and a maximum number of locations where a product must be delivered.

21. The procurement manager of Claim 19, wherein the option comprises a plurality of ranges of parameters each selected from a group consisting of:

a minimum quantity of a product that the buyer is obligated to purchase, and a maximum quantity of the product that the seller is obligated to supply;

a minimum number of product types that the buyer is obligated to purchase, and a maximum number of product types that the seller is obligated to supply; and

a minimum number and a maximum number of locations where a product must be delivered.

22. The procurement manager of Claim 19, wherein the option contract includes an exercise period comprising a period of time after the execution of the option contract during which the buyer must exercise its option, and wherein the exercise module is further operable to specify a first quantity of product desired at a first time during the exercise period and to specify a second quantity of product desired at a second time during the exercise period, the buyer's obligation under the option contract comprising the sum of the first and second quantities of product desired.



23. The procurement manager of Claim 19, wherein the negotiating module is further operable to receive a proposed contract term from the seller computer, access a memory comprising a range of acceptable contract terms, determine that the proposed contract term is within the range of acceptable contract terms, and to accept the proposed contract term without user input.

24. The procurement manager of Claim 19, wherein the negotiating module is further operable to receive a proposed contract term from the seller computer, access a memory comprising a range of acceptable contract terms, determine that the proposed contract term is not within the range of acceptable contract terms, and to identify the proposed contract term as a term requiring user input prior to acceptance.

25. The procurement manager of Claim 19, further comprising an aggregation module operable to compare a buyer's aggregation of parameters with a seller's aggregation of parameters, and to transform at least one of the aggregations of parameters to conform with a common aggregation of parameters.

26. The procurement manager of Claim 19, further comprising an option price module operable to determine a proposed option price comprising a value of the option to a buyer associated with the procurement manager and to communicate the proposed option price to a seller computer, and wherein the negotiation module is operable to negotiate with the seller computer an agreed option price based on the value of the option to the buyer and a cost of the option to a seller associated with the seller's computer.

27. The procurement manager of Claim 19, further comprising a tracking module operable to store terms of the executed option contract and to track the buyer's fulfillment of its obligations under the option contract.

28. A supply manager operable to be executed on the processor of a seller computer, the supply manager comprising:

- a forecast module operable to determine the seller's range of forecasted supply capacity for a product;

- a negotiation module operable to receive from a buyer computer an offer to enter into an option contract for the supply of a product, the option contract including a proposed option corresponding to a range of forecasted demand;

- an execution module operable to execute the option contract and to store the terms of the option contract in a memory accessible to the seller computer; and

- a tracking module operable to receive, from the buyer computer, a request to exercise at least a portion of the option; and

- in response to receiving the request:

- access the stored terms of the option contract; and

- using the stored terms of the option contract:

- determine whether an option period specified in the option contract has begun;

- if the option period has not yet begun, notify the buyer computer that the request is premature; and

- if the option period has begun:

- determine whether the request specifies a request quantity that exceeds a maximum option quantity specified in the option contract;

- if the request quantity exceeds the maximum option quantity, notify the buyer computer that the request is improper; and

- if the request quantity does not exceed the maximum option quantity, storing the request system for seller compliance.

29. The supply manager of Claim 28, wherein the option comprises a range of parameters selected from a group consisting of:

a minimum quantity of a product that the buyer is obligated to purchase, and a maximum quantity of the product that the seller is obligated to supply;

a minimum number of product types that the buyer is obligated to purchase, and a maximum number of product types that the seller is obligated to supply; and

a minimum number and a maximum number of locations where a product must be delivered.

30. The supply manager of Claim 28, wherein the option comprises a plurality of ranges of parameters each selected from a group consisting of:

a minimum quantity of a product that the buyer is obligated to purchase, and a maximum quantity of the product that the seller is obligated to supply;

a minimum number of product types that the buyer is obligated to purchase, and a maximum number of product types that the seller is obligated to supply; and

a minimum number and a maximum number of locations where a product must be delivered.

31. The supply manager of Claim 28, wherein the option contract includes an exercise period during which the buyer must exercise its option, and wherein the option contract comprises a maximum supply quantity that the seller has agreed to supply, and wherein the request to exercise the option comprises a first request for a first quantity of product desired, and wherein the tracking module is operable to store the request in the memory if a current date is within the exercise period and the first quantity is less than or equal to the maximum supply quantity.

32. The supply manager of Claim 31, wherein the request to exercise the option comprises a second request for a second quantity of product desired, and wherein the tracking module is operable to store the request in the memory if a current date is within the exercise period and the sum of the first and second quantities is less than or equal to the maximum supply quantity.

33. The supply manager of Claim 28, wherein the option contract comprises a penalty term specifying a penalty for a violation of the contract terms, and wherein the tracking module is operable to identify a violation of the contract terms and to assess a penalty for the violation based on the penalty term.

34. The supply manager of Claim 28, wherein the offer to enter into the option contract comprises a proposed term, and wherein the negotiation module is operable to access a memory comprising a range of acceptable contract terms, determine that the proposed contract term is within the range of acceptable contract terms, and to accept the proposed contract term without user input.

35. The supply manager of Claim 28, wherein the offer to enter into the option contract comprises a proposed term, and wherein the negotiation module is operable to access a memory comprising a range of acceptable contract terms, determine that the proposed contract term is not within the range of acceptable contract terms, and to identify the proposed contract term as a term requiring user input prior to acceptance.

36. The supply manager of Claim 28, further comprising an aggregation module operable to compare a buyer's aggregation of parameters with a seller's aggregation of parameters, and to transform at least one of the aggregations of parameters to conform with a common aggregation of parameters.

37. The supply manager of Claim 28, further comprising an option price module operable to determine a proposed option price comprising a cost of the proposed option to a seller associated with the supply manger and to communicate the proposed option price to the buyer computer, and wherein the negotiation module is operable to negotiate with the seller computer an agreed option price based on the value of the option to the buyer and a cost of the option to a seller associated with the seller's computer.

38. The method of Claim 1, further comprising, at the buyer computer:  
monitoring an exercised portion of the option;  
comparing the exercised portion of the option with a minimum option quantity specified in the option contract to determine whether a buyer obligation under the option contract has been met;  
determining whether an option period specified in the option contract has ended; and  
if the buyer obligation has not been met and the option period has ended, determining an applicable buyer penalty based at least in part on a penalty specified in the option contract.

39. The procurement manager of Claim 19, further comprising a tracking module operable to:  
monitor an exercised portion of the option;  
compare the exercised portion of the option with a minimum option quantity specified in the option contract to determine whether a buyer obligation under the option contract has been met;  
determine whether an option period specified in the option contract has ended; and  
if the buyer obligation has not been met and the option period has ended, determine an applicable buyer penalty based at least in part on a penalty specified in the option contract.

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# PURCHASING HANDBOOK

*Standard Reference Book on Purchasing  
Policies, Practices, Procedures,  
Contracts and Forms*

GEORGE W. ALJIAN

*Editor-in-Chief*

VICE-PRESIDENT OF PURCHASING, PACKAGING, AND  
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## PURCHASING HANDBOOK

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- Contributions by these  
and have been revised, re-  
stated professional position  
or her contribution.

*Inspection as a Check on Specifications.* All specifications must be reasonably checked. Inspection is not a function of the purchasing department, but the purchasing department should be informed of results so that it can place orders with the sellers who supply the most satisfactory goods and services.

The purchasing department should make certain that the first castings from a new pattern or mold, the first stampings from a new die, or the first shipment of materials made to blueprints or specifications are checked and approved by engineering, production, or other authority before production quantities are released.

The buyer should show on purchase orders for patterns, tools, dies, molds, jigs, and fixtures that "Payment will not be approved until production samples have been inspected and approved by purchaser." The purchase order should also state who owns, maintains, and insures these special production facilities.

*Savings through Specification Examination.* One additional method used to promote money savings is to continually examine specifications from which production, maintenance, and operating materials are purchased. A study of such specifications may result in modifications wherein the function performed will not be altered but its cost will be reduced. It is important to determine the origin of specifications currently in use. A thorough investigation by competent purchasing personnel might indicate that specifications should be modernized to include newer, improved materials.

#### *Buying Proper Quantity, Section 8*

Buying proper quantity usually includes a formula for purchasing minimum and maximum quantities and a typical section on this subject follows:

**BUYING PROPER QUANTITY:** The quantities to be bought are determined from known factors of demand, supply, and cost. The demand factor appears from the estimates of the sales, production, or maintenance departments, or from the records of the stores department showing withdrawals from stock. The supply factor is drawn from the reserve stock of any item carried in inventory and from the time required to secure delivery. The cost factor is derived from the inventory carrying charges and the price advantages possible on quantity purchases. In order to establish the quantity to be bought, these factors must be combined in such a manner that the material will cost the least in terms of ultimate cost.

Since the most economical quantity to order at any one time seldom meets the requirements of demand, some storage of materials is desirable

from an ultimate cost materials will be available.

It is not usually a the demand. This scheduling, or stores should continuously anticipated changes which determine quantity department to determine the proper quantities

*Formula for Minimum* ing formula is: immediate demand divided by safety.

This formula makes order, possible saving in market prices. Minimum ordering quantity will larger quantity, or with of a larger order would

*Optimum Ordering* a point where the cost obtained by ordering is

A number of formulas published in purchasing books (as well as in other considered are:

1. The cost of procurement until the goods have in stores. This includes expense of the purchase that of the accountment.
2. The quantity to be purchased
3. The interest charges
4. The storage charges
5. The reserve stock needed
6. The unit purchase price

<sup>4</sup> See Sections 13 and 29.



from an ultimate cost standpoint. Storage is also necessary to ensure that materials will be available when and where they are needed.

It is not usually a function of the purchasing department to determine the demand. This must be decided by the manufacturing, planning, scheduling, or stores departments. The purchasing department, however, should continuously keep informed about operating conditions and any anticipated changes which will affect demand. The other two factors which determine quantity—availability and cost—are for the purchasing department to determine. It is, therefore, logical that the interested departments need to cooperate to assemble all the factors and determine the proper quantities to order.

*Formula for Minimum Ordering Quantity.* Stated simply, a good working formula is: immediate net demand, minus stock on hand, plus the estimated demand during the delivery period, plus a reasonable margin of safety.

This formula makes no allowance for a possible minimum charge per order, possible savings by ordering a larger quantity, or for fluctuations in market prices. Minimum ordering quantity should be used as an actual ordering quantity when it is as economical to buy this amount as any larger quantity, or when a drop in prices is probable, or when storage of a larger order would be impractical.

*Optimum Ordering Quantity.* This quantity should be established at a point where the carrying costs and risks on inventory balance the gains obtained by ordering in larger quantities.

A number of formulas for determining the proper quantities have been published in purchasing, inventory control, and production control textbooks (as well as in other sections<sup>4</sup> of this handbook). Some of the factors considered are:

1. The cost of procurement from the time the purchase is requisitioned until the goods have been received, checked, inspected, and placed in stores. This includes the expense of receiving and inspecting, the expense of the purchasing department chargeable to the order, and that of the accounting department in handling the invoice and payment.

The quantity to be purchased for a given period.

The interest charges for carrying the inventory.

The storage charges on the goods.

The reserve stock necessary for emergencies.

The unit purchase price.

<sup>4</sup>Sections 13 and 29.

companies could apply such a system to between 80 and 90 per cent of all purchase orders issued and would find that this volume of paper work covers approximately 15 per cent of the purchasing-dollar commitments.

## BLANKET PURCHASE ORDERS

A blanket order is *incomplete* in that it is not usually an authorization to ship anything. Only when the specified vendor receives a bona fide shipping requisition or other type of release, written or verbal, does the act of purchasing become effective.

Blanket orders may cover nearly every type of material from chemicals and inks to bread and window-washing services. Reasons for using blanket orders and the typical manner in which they assist in the purchasing function are:

1. Reduce paper work on those items bought repetitively.
2. Provide a means for obtaining the advantages of quantity purchasing power. The total quantity representing an organization's purchasing power on a given item might come from combining the requirements of various plants or from merely determining the total requirements for a given year at a single facility.
3. Decentralize the actual act of ordering materials on a day-by-day basis while maintaining centralized control over the selection of vendors and the establishment of prices, terms, and other conditions.
4. Reduce the amount of routine information exchanged through the purchasing department after an order has been placed. Since all terms, prices, and conditions are present, the feeling here is that the physical

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TERMINATIONS:  
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e shown in Fig. 25-4b.

- checking of invoices and the receiving of information become an accounting responsibility.
5. Release professional purchasing people from the tasks of handling repetitive, routine transactions. By covering such transactions in the mechanics of the blanket order, purchasing agents and buyers can then concentrate their efforts on nonroutine acquisitions, with only periodic reviews of the various blanket orders.
  6. Other:
    - a. Provide price protection for a specified period of time.
    - b. Minimize in-plant inventories. Vendors receiving blanket orders are expected to maintain an inventory on the items covered.
    - c. Provide organized procedure for reviewing purchases of groups of commodities on a regular basis.

Under any blanket-order system the procedure for establishing and instituting the form should be such that purchasing will maintain its basic responsibility for providing materials as needed and controlling the dollars spent for them. This control need not be lost when purchasing delegates authority to another department to issue releases directly to the supplier. One copy of the release form should be sent to purchasing, enabling the department to maintain a clear record of the purchases throughout the life of the order. Normally, blanket orders should not be used for items involving large sums of money if the needs and schedules can be rather accurately determined. Maximum benefits are achieved when they are used to eliminate a large number of rush and routine small dollar-value transactions.

### BIBLIOGRAPHY

Same as for Section 4, "Legal Influences in Purchasing."

Other books on purchase orders are listed in Section 26, "Library and Catalog File."

Additional samples of purchase order forms are illustrated in Section 25, "Forms and Records."

ETI

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### PURPOSE OF INVENTORY

Inventory is created for two general purposes, namely, protection and economy:

1. To provide sufficient material to meet demands for the particular raw material, fabricated part, or finished product with a minimum of delay (i.e., protection)
2. To effect lower product costs by realizing the economies resulting from longer manufacturing runs, and from purchasing larger quantities per order (i.e., economy)

Although there may be a variety of individual reasons for creating inventory under a given set of circumstances, if closely examined, they will all fall into one of the two categories mentioned above.

### FACTORS INFLUENCING INVENTORY

In the act of creating inventory, there are two fundamental questions which must be answered in every instance, namely:

1. How much to buy (or manufacture) at one time.
2. When to buy (or manufacture) this quantity.

Answers to these two questions must come from a proper consideration and evaluation of a number of different factors having a bearing on inventory and its control. Four of these factors are fundamental, and, without consideration for costs, could form the basis for any decision on how much to buy and when to buy. These are as follows:

1. *Requirements*, or demand, on a unit/time basis. This is based upon information from a production or sales forecast schedule.
2. *Quantity in stock and on order*. This is usually obtained from a ledger record showing a stock balance plus any open purchase or manufacturing orders for previously known requirements.
3. *Procurement time*, or lead time, is the total length of time to obtain a fresh supply of the item.
4. *Obsolescence*. Consideration must always be given to the possibility of design changes or other factors which would make the material obsolete.

If purchasing or manufacturing is to be done strictly to requirements, consideration of the above factors will be sufficient to determine "how much" and "when" to order. However, if attention is to be given to re-

ducing inventory costs, then the following considerations must be added to the above list; namely:

1. *Ordering cost.* This includes the cost of processing the purchase order, receiving and inspection costs, freight charges, and accounts payable costs to pay the vendors' invoices.
2. *Inventory carrying cost.* This factor includes interest rate on average dollars in stock, insurance, cost of depreciation and obsolescence, and cost of storage facilities (rent, light, heat) based upon floor space of storage area, taxes, and handling costs.

While the application of these cost factors to specific control techniques will be discussed in more detail later in this section, it should be noted at this point that the proper evaluation of the ordering and carrying costs is the key to one of the largest areas for reducing cost by inventory control.

It should be readily understandable that the ordering cost, or carrying costs, will vary from industry to industry, and even between two plants in the same industry. However, the determination of these cost factors is a "must" if real "cost control" of inventory is to exist. Since these costs are the direct result of the frequency of ordering and size of inventory, proper evaluation and comparison of the ordering and carrying cost under various ordering frequencies will reveal a most economical frequency or size of order, as will be discussed later in this section.

#### *Ordering Cost (Variable)*

Computation of this cost should be based on a period of not less than one year.

Accuracy in establishing the cost elements needed to arrive at the ordering cost is essential to sound inventory policies. Determining the "fixed" and "variable" portions of this cost should be the result of careful consideration.

The following data are required:

1. Total number of purchase orders issued.
2. Labor costs (and payroll costs) for the entire purchasing department, material control, receiving, and stores. Some of these costs are fixed, whereas some will vary with the number of orders issued. For example, the purchasing agent and the senior buyers would be "fixed costs"; other purchasing department personnel would be "variable." Stockroom personnel are charged either to the cost of carrying inventory or to the cost of ordering. (Usually these costs are 75 per cent for carrying inventory and 25 per cent for ordering.)

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3. Cost of supplies (expense items). These include printed forms, paper, ribbons, pencils, etc. About half of these costs would be considered variable.
4. Floorspace and maintenance. Cost of office space occupied by purchasing and material control. About 75 per cent is variable.
5. Incoming inspection. Cost of inspection personnel, of which about 25 per cent is variable.
6. Accounts payable. Labor and payroll costs. About 90 per cent variable.
7. Inbound freight. About 25 per cent of this cost item is considered variable.
8. Telephone and telegraph. About 50 per cent of these costs incurred by buying personnel are variable costs.

Once all the above cost elements have been assembled and divided between fixed and variable costs, tabulation like the following can be made.

## EXAMPLE

Expenditure	Actual cost	% of actual cost	Variable cost
Labor .....	\$400,000	60	\$240,000
Supplies .....	18,000	50	9,000
Space—occupancy .....	40,000	75	30,000
Incoming inspection .....	100,000	25	25,000
Accounts payable .....	20,000	90	18,000
Inbound freight .....	100,000	25	25,000
Telephone and telegraph .....	16,000	50	8,000
Total .....	\$694,000		\$355,000

Total number of purchase orders for the year: 20,000

Actual cost per purchase order:  $\frac{\$694,000}{20,000} = \$34.70$

Variable cost per purchase order:  $\frac{355,000}{20,000} = \$17.75$

Average number of items per purchase order: 2.5

Variable cost per item:  $\frac{\$17.75}{2.5} = \$7.10$

*Inventory Carrying Cost (Variable)*

Accuracy and careful consideration are again of primary importance. The following data are needed to establish the cost of owning inventory:

1. Average monthly inventory for the period (year) under review.
2. Interest. This is the current bank rate, charged for the use of money (4 to 8 per cent would seem about right). Variable 100 per cent.

3. Taxes. This may vary from state to state, but will probably be 2 to 3 per cent of average inventory. Variable 100 per cent.
4. Insurance. This, too, will vary, with a figure of 10 cents per \$1,000 probably close to most rates. This, too, is a 100 per cent variable cost.
5. Obsolescence. Actual amounts of purchased material, written off, as obsolete, are used here. Normally this would be between 2 and 8 per cent, and it is 100 per cent variable.
6. Shrinkage. Where this is a factor, allowance should be made for it; 1 per cent seems about the permissible maximum, and about 20 per cent of the cost would be variable.
7. Labor costs. These costs were used above in determining the cost of ordering, except for a portion of stockroom personnel. Stores people will "vary" in accordance with factory requirements (manufacturing orders), and the cost is not considered variable with inventory levels.
8. Floorspace and maintenance. The floorspace and other costs for the stores areas are usually variable 90 per cent, with 10 per cent fixed costs. This must be determined separately for each plant operation.
9. Scrap. Actual amount of scrapped (purchased) materials is a variable cost. This would normally be less than 0.5 per cent of the average inventory.

After assembling and apportioning the above cost elements, a tabulation, similar to the following, can be made.

## EXAMPLE

Expenditure	Actual cost	% of actual cost	Variable costs, %
Interest on investment .....	\$144,000	100	6
Taxes .....	72,000	100	3
Insurance .....	240	100	0.01
Obsolescence .....	144,000	100	6
Shrinkage .....	2,400	20	0.02
Labor .....	70,000		
Space—occupancy .....	80,000	90	5
Scrap .....	7,500	100	0.31
Total .....	\$520,140		20.34

Average monthly inventory: \$2,400,000

Variable cost (per cent) of carrying inventory = 20.34%\*

Variable cost of carrying inventory: \$2,400,000 × 20.34% = \$488,160

\* See Table 29-16 for a composite of estimates involved in carrying charges for inventory.

OF

In its broadest the creation of It encompasses the

1. Forecasting future
2. Characteristic
3. Organization

*Forecasting Future*

*Forecasting.* In forecasting a serious inventory, the forecasting of future production and cost of factory inventory tends to be a very serious cost. It is important to

1. The inventory requirements.
2. Attempts to goods fluctuate
3. Inaccurate sales insufficient in Fig. 13-1.

These considerations between the sales emphasize the importance to current

*Inventory Policy.* Inventory policy, timely warning control system in materials use the inventory use as a "barometer" the ratio of sales ratio is of significance hand, and open

## ORGANIZING FOR INVENTORY CONTROL

In its broadest sense, effective inventory control requires more than just the creation of a control organization and delegation of responsibilities. It encompasses three general areas of study, namely:

1. Forecasting future requirements and inventory turnover
2. Characteristics of the inventory and classification of accounts
3. Organization of manpower and delegation of responsibility

*Forecasting Future Requirements and Inventory Turnover*

*Forecasting.* If an objective survey were made of companies experiencing a serious financial condition today due to excessive investment in inventory, the dominant contributing factor would be inaccurate forecasting of future sales. The accuracy of sales forecasts as translated into production and inventory requirements decidedly affects cost of inventory and cost of factory operations. Investigation will prove that excessive inventory tends to go hand in hand with a short cash position—a particularly serious condition for any company with limited working capital. It is important to be cognizant of the following relationships:

1. The inventory level is directly related to sales and production requirements.
2. Attempts to change production rates as frequently as sales of finished goods fluctuate will result in excessive factory costs.
3. Inaccurate sales and production forecasts will result in excessive or insufficient inventory, plus the associated extra costs which are shown in Fig. 13-1.

These considerations highlight the need for a close working relationship between the sales department and inventory-control organizations, and emphasize the need for a flexible inventory plan capable of quick adjustment to current conditions.

*Inventory Turnover.* The inventory manager may well ask, "Is the inventory policy keyed to a 'barometer' that will provide adequate and timely warning of a change in the economic picture? Does the inventory-control system provide a means for detecting future increases or decreases in materials usage in time to allow corrective measures for adjusting the inventory level to meet the changing picture?" Many companies use as a "barometer" inventory movement or turnover, which is simply the ratio of sales to stocks on hand. The method used to develop the ratio is of significance; it should incorporate sales forecasting, stocks on hand, and open commitments, as well as a comparison of past inventory



movement. A general discussion of the techniques applied is advisable. To find turnover of materials in manufacturing, one may take the requisitions during the period and divide by the average materials inventory. This will give an index of the movement of materials independent of the production process. The turnover of materials may also be computed in terms of production process by using the material component of goods completed. However, it is most normal to use the disbursements from stock, or usage as it may be called, as the basis of arriving at the number of months' inventory in stock. The work-in-process turnover may be computed by dividing total production cost by the average inventory of work in process. The turnover of finished stock may be computed as purchased merchandise, that is, the average inventory of materials, work in process, and finished goods would be divided into the cost of goods sold in order to arrive at a rough measure of operating inventory turnover.

*Frequency of Turnover.* How many times should an inventory turn over per year? Surveys have indicated that industry tends to average approximately ten stock turnovers annually, but this figure differs widely in the type of business. For example, the retail dairy and poultry products have a turnover of 36. Meat markets operate at a turnover of 38. Fur shops manage to make a profit in turning their inventory only twice a year. Let us recognize, however, that there is a danger in being over-aggressive in increasing the frequency of turnover, since this can lead to serious consequences. Elaborating, we may say, turnover is not the cure-all that some consider it to be. Nor do all inventory rules go by the board when the ratio of turnover to investment is boosted, and when the number of annual stock turns increases, total costs decline and profits rise—but only to a certain point. Beyond this point in many instances, the reverse occurs. The nature of the industry and the unique conditions in individual companies are the final determining factors. It is important to emphasize that inventory turnover in terms of total inventory may be greatly misleading and one must break it down into classes relating the impact of frequency of turnover to operations and net profits in order to value the movement more concisely.

To illustrate, in one firm under review, the inventory level stood at 1.7 months on hand with a turnover of approximately seven times a year. On the surface, this looked excellent, but this firm was suffering a substantial loss due to parts shortages. It is necessary, therefore, that the degree of inventory turnover be consistent with the tolerable number of shortages that would not adversely affect production. As a general rule, a turnover of four to six times a year in the manufacturing and servicing types of industries may be considered acceptable.

*Characteristics of the Inventory and Classification of Accounts.* One of the first steps in organizing for more effective inventory control, and perhaps the key to ultimate success of a control plan, is the making of an analysis, tabulation, and classification of the characteristics of the commodities being carried in inventory. The thoughtful classification of accounts will permit sound financial control of inventory, by directing the attention of the inventory planner to those accounts which control the majority of dollars. Some of the more important points of consideration in making such a classification or analysis of commodities are:

1. *Classification by Usage.* An analysis of usage (sometimes called activity) has been found to be most useful when prepared in the form of a listing of commodities in order of descending dollar activity, this is sometimes expressed as the ABC concept. This list, when totaled in cumulative fashion, will indicate at a glance which accounts control the bulk of dollars in inventory. It is of interest to note that the curve shown in Fig. 13-2 is typical of a manufacturing inventory. Approximately 10 per cent of the items stocked account for 70 per cent of the dollar investment; at the other extreme, 70 per cent of the items represent only 10 per cent of investment, with the remaining 20 per cent of the items representing 20 per cent of the investment. This relationship is shown in bar chart form in Fig. 13-3.

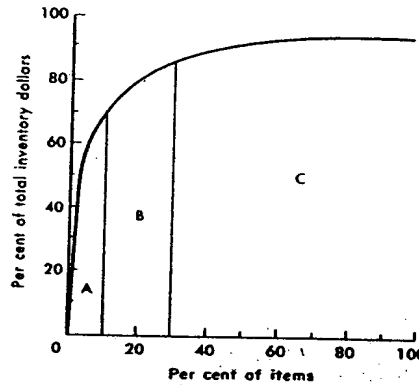
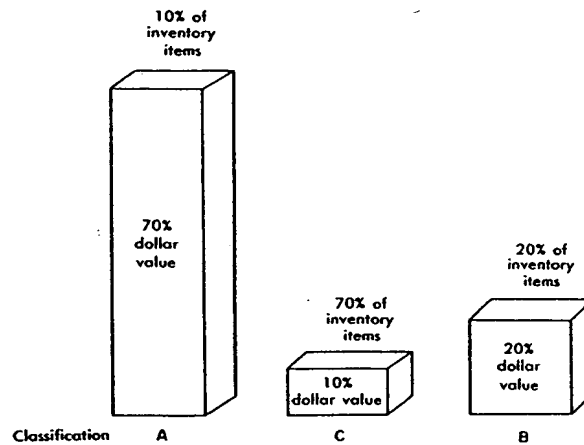


FIG. 13-2. Distribution of inventory items by inventory dollars.

Here, then, is one of the keys to better inventory control. The control policies and techniques which apply to the few items representing 70 per cent of the total inventory value (classification A) would be designed for close supervision, through continual review of requirements, stock balances, and scheduled materials deliveries to maintain a minimum of inventory. In the automotive and consumer appliance industries, it is common practice to schedule these commodities in such a manner that the factory uses the material right out of the railroad car or delivery truck. The only inventory which exists, in this case, is that which is in transit from the supplier to the factory.

The 70 per cent of inventory items (classification C) which comprise the balance of 10 per cent of dollars may be controlled by the maximum-minimum method, by a statistically developed formula which

provides a "protective" stock, or some other means which creates an economical ordering frequency. The reordering and control of these items should be reduced to an automatic basis, such as "stockless" purchasing under long-term contracts or blanket purchase orders, described in Section 5.



Classification A. Controlling items for inventory investment.

Maintain conservative stock level.

Constantly review and adjust scheduled open commitments.

Classification C. Avoid shortages by maintaining adequate stock levels with low risk due to minimum investment. Reorder stock on automatic basis.

Classification B. Maintain adequate stock levels with scheduled deliveries and periodic reviews of key items.

FIG. 13-3. ABC selective technique of inventory control.

This classification of accounts by activity is a tool which can be applied to the items in inventory as a whole, as suggested above, or to the inventory items in one selected commodity. In either case, the important result is that such an analysis permits a ledger controller to devote his time to "controlling" those items which significantly affect inventory dollars.

In addition to the savings which can be effected by applying special ordering and scheduling of the high dollar value inventory accounts, an analysis of the type mentioned above will also enable the best application of efforts for reduction of costs in other areas of inventory control, such as transportation, storage facilities, and material handling.

2. Transportation Analysis. In connection with transportation, for example, the particular problems of those accounts affecting the bulk of

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inventory and incoming freight expense should be given special study by the inventory controller and the traffic department. Techniques such as the consolidation of shipments into truckloads from a given area, establishment of special rates with one trucking company based on a sustained volume, reduction of the number of different truck lines being used by suppliers, improved in-transit packaging for reduced loss due to damage, and others are all ways of obtaining a more efficient inventory operation with reduced costs.

3. *Storage and Handling Analyses.* Likewise, in the area of storage and material handling, attention should be pinpointed to those items involving the largest volume to be stored or handled. Such things as special packaging to permit easier handling or reduction of storage space, better utilization of "air space" by stacking or use of racks, establishment of a unit package as received from the supplier containing a specified quantity to accommodate the normal factory requirement for a given period, and conveyerization within storeroom and from storeroom to the area of use are just a few of the many profit-making ideas which should be given study.

*Organization of Manpower and Delegation of Responsibility.* In the field of inventory management, there is no standard of optimum organization which can be applied to any industry or company. Because inventory control has an effect upon so many different phases of a business enterprise, the responsibility for inventory management and organizational structure to discharge this responsibility may take different forms. In some companies the purchasing department is charged with this responsibility; in others, the production department handles it. In still others, the control is divided between purchasing and production according to commodity. In recent years, many large companies have set up separate inventory planning and control organizations or committees, with responsibility for establishing policies and practices. This committee should include representation from the accounting, purchasing, production, engineering, and sales departments.

Fundamentally, the organization for inventory control should be established with careful regard for four basic considerations:

1. The heavy influence which inventory control has upon all departments of the company.
2. Qualifications of inventory-control personnel, particularly the person to be charged with over-all responsibility.
3. Need for flexibility in the systems and control measures established.
4. Importance of communication. Regardless of the organizational assignments with respect to inventory control, there must be open channels

of communication for the free and prompt exchange of information relative to inventory.

A few of the large manufacturing concerns have solved this "communication" problem by establishing a "manager of materials" with line responsibility over the purchasing, material planning, stores, receiving, and traffic departments. This centralization of responsibility facilitates the accomplishment of a planned and coordinated program of inventory control.

Of all the departments having an interest in inventory control, the purchasing department is probably the most directly affected by, and in turn, can most influence, the inventory control measures which are applied. The purchasing agent, or director of purchases, almost without exception, should participate strongly in the establishment of inventory policies regardless of the extent of his direct responsibility for inventory control. Three degrees of purchasing participation might, therefore, be considered:

1. Purchasing agent is directly responsible for control of stores. Delegation of stores control to the purchasing agent has a number of advantages among which are:
  - a. Communication problems reduced to a minimum.
  - b. Highest degree of flexibility for changes in production rate or inventory level.
  - c. Opportunity for integration with suppliers with respect to combined inventory investment.
  - d. Optimization of over-all acquisition and inventory cost.
  - e. Direct responsibility for stores shortages and their control. This places purchasing in a key position in the manufacturing cycle, through expeditious handling and minimizing lost production time due to shortages.
2. Purchasing helps establish policies. Under this type of operation, the purchasing department would be a member of an inventory-control committee. This committee would establish policies on such matters as inventory standards, sales forecasting, load planning, etc., the formulation of which requires data and analysis which only purchasing can logically furnish. The importance of having a well-qualified purchasing representative is self-evident.
3. Purchasing has no responsibilities. Even though purchasing may not be a part of the inventory policy-making group, the responsibility still prevails for formulating a purchasing program based on sound inventory control. Bargaining position, procurement efficiency, long-term favorable relations with suppliers, maximizing profit opportuni-

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ties cannot be attained without effective inventory planning and control on the part of the cognizant department.

Table 2-7, Section 2 shows the purchasing department responsible for inventory control in the majority of cases. However, it should be considered that in some cases purchasing may only play a minor role in formulating inventory policy with little or no responsibility for setting the level of investment, being relegated only to the clerical task of "keeping the ledgers." Real responsibility for the inventory-control function exists when purchasing establishes the concepts and devises the system and procedure for control. Among many criteria the size of the company, the kind of business, the types of commodities, the nature of the system, and the degree of control required by management are each determining factors in establishing the place of purchasing in inventory management.

### FUNDAMENTALS OF ORDER POINT AND ORDER QUANTITY

Inventory is created for two reasons:

1. To reduce over-all operating costs (economy)
2. To provide protection against unpredictable demands (protection)

It is usually difficult to isolate these two characteristics when examining a given stock account, but it is necessary to do so if effective methods of inventory control are to be installed. The following sections will illustrate the economy function alone, then the economy and protection functions combined, and finally, the protection function alone.

#### *Economy Function*

An inventory of purchased raw material and fabricated parts may be created even though the company in question could operate with no inventory at all. For example, assume that X Company makes a product whose sales can be predicted, item by item, for a period of a year or more (or whose production schedule is firm for a year, and the finished products which are not sold are put into finished stock inventory). In this situation, each product could be "exploded" into all the required raw material and parts, and the requirements of each piece of purchased material could be pinpointed to the exact day it would be needed. All material could be brought in daily and routed directly to the manufacturing area where it was needed. The only inventory would be in-process inventory. Assuming that X Company has 5,000 different items which are purchased regularly, such a policy would mean scheduling hundreds of

incoming shipments daily, with all the associated paper work, checking, and following. Therefore, X Company would quickly conclude that it would be more economical to buy, receive, and store these items *in advance of need*. They would accomplish this by creating raw material stock accounts. The creation of this raw material inventory would, of course, result in additional costs associated with storing the material, obsolescence, depreciation, etc. These so-called inventory carrying costs, when compared with the previously mentioned costs associated with ordering and receiving, may be used to calculate the most economical policy of ordering and storing the inventory. To repeat, inventory would be created in advance of its actual need, and held for weeks or perhaps months, because it would prove cheaper to carry that inventory for that period of time, rather than incur the costs of daily or weekly ordering and/or receiving.

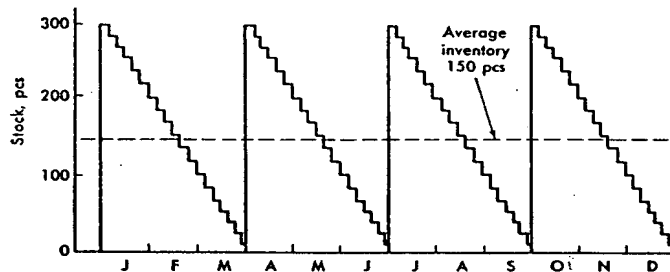


FIG. 13-4. Predictable monthly usage—100 pieces order quantity—300 pieces.

Figure 13-4 shows the rise and fall of the stock level for a given stock item whose predictable usage is 100 pieces per month. Assume that for this particular item the most economical method of ordering<sup>2</sup> is in quantities of 300 pieces, once every three months. The stock controller would place his orders so that each new order would arrive just as the stock reached zero. The stock level would fluctuate between 300 and zero, resulting in an average inventory of 150 pieces, or  $1\frac{1}{2}$  months' supply.

The inventory which is created by the use of a predetermined order quantity is directly related to that order quantity, provided the same quantity is used on all subsequent orders. Figure 13-4 shows this to be true:

$$\text{Average active stock} = \frac{1}{2} \text{ order quantity}$$

In this example, the inventory is composed entirely of active stock, that is, every piece of stock "turns over" during the period between receivings.

<sup>2</sup> See calculation under "Using the Economical Order Quantity," page 13-26.

Applying this principle means for forecasting one-half times the

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#### Protection Function

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<sup>3</sup> This neglects Stock," page 13-21

## FUNDAMENTALS OF ORDER POINT AND ORDER QUANTITY 13-17

Applying this principle to every item in the inventory, it provides a means for forecasting the total inventory. The total inventory would be one-half times the sum of each order quantity.<sup>3</sup>

This illustration is oversimplified to illustrate how the active stock portion of the inventory is created. Except for companies whose production schedules are 100 per cent predictable, most raw material inventories must contain an additional segment of inventory, called "safety stock." This subject is discussed next.

### *Protection Function (Safety Stock) and Economy Function*

A more typical situation than the case of X Company is found in Y Company. The latter organization manufactures a product which can be forecasted only to general product lines. The individual items within those product lines may be required by its customers upon short notice.

January.....	970
February.....	850
March.....	1,010
April.....	900
May.....	910
June.....	980
July.....	1,770 (maximum monthly usage)
August.....	910
September.....	830
October.....	980
November.....	910
December.....	980
	<hr/> 12)12,000
	1,000 (average monthly usage)

FIG. 13-5. Stock issue record.

This short lead time does not allow time enough to purchase the material for each individual customer's order. Therefore, there is an obvious need for the creation of raw material inventory, to be available for these periodic demands. The stock level of each stock item may be set initially by guess or intention, but after a period of months, the issue pattern from the stock ledger should provide for a more factual approach. That is, past activity may be used to forecast probable future requirements. An example of such a historical record is shown in Fig. 13-5.

This example is shown to illustrate two points:

1. If the item in question is a stable item, used frequently, so that many demands are made upon it each month, there will be an issue pattern

<sup>3</sup> This neglects the "safety stock" portion of the inventory. See "Calculation of Safety Stock," page 13-29.



- which may be used to forecast future activity. This rate of usage may be expressed as "average monthly usage," or usage per any convenient time period.
- 2. Even the most stable of the items will occasionally experience a period of unusually high activity. This may be called "maximum monthly usage," and may exceed the "average monthly usage" by a significant amount.

Figure 13-5 illustrates an item whose "maximum monthly usage" (1,770 pieces) is almost twice its "average monthly usage" (1,000 pieces).

Lacking any other means to forecast his future requirements, the controller of this stock item would do two things:

1. He would establish a pattern for reordering his stock so that he would receive about 12,000 pieces during the following year.
2. He would select an order point which was sufficient to cover the expected usage during the reorder period.

Using the data from Fig. 13-5, he might do the following:

1. Use an order quantity of 2,000 pieces, every two months.
2. Assuming a supplier's lead time of one month, use an order point of 1,000 pieces.

If this policy were followed, the stock level might follow a pattern as shown in Fig. 13-6a. As long as the monthly demands on the item did not exceed 1,000 pieces, the stock would be adequate to cover these demands. But as soon as a high activity month occurred (August), the stock would reach zero before the next order arrived. A stockout period of a week or two could result.

This policy, then, must be adjusted so that some extra stock is provided in the system to cover these occasional periods of maximum usage. This is done by raising the order point, and hence the stock level, as shown in Fig. 13-6b. The order point should be based on the *maximum* expected usage, rather than the *average* usage during the lead-time period. In this example, the lead time is one month. The maximum expected usage might be arbitrarily set at 2,000 pieces, and this would be used as the order point. The result of this policy is to create additional stock, to be used only during periods of above-average usage. This is called the "safety stock" portion of the inventory. Once established, this safety stock need not be reordered. This is true because each period of high activity would be offset by periods of below-average activity, which would tend to keep the safety stock at its original level. In this example, then, the order quantity would remain unchanged, at 2,000 pieces, every

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## FUNDAMENTALS OF ORDER POINT AND ORDER QUANTITY 13-19

two months. The stock level would not drop below the safety stock level (1,000 pieces) until the high activity during August would deplete it temporarily. The stockout period shown in Fig. 13-6a would be prevented by the introduction of safety stock (Fig. 13-6b).

NOTE: The rise and fall of the stock level, on the typical stock item, is not as symmetrical as the ones illustrated, which have been deliberately oversimplified. The safety stock portion of the inventory is therefore difficult to visualize. It may be measured, however, by simply measuring the points of minimum stock over a period of months, and calculating their average value. The active stock portion of the inventory is then calculated by subtracting the safety stock from the average inventory over that period.

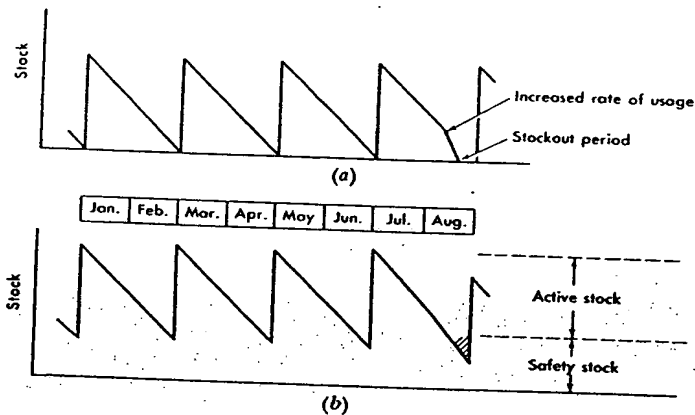


FIG. 13-6a and b. Stock flow and replenishment chart.

It should be emphasized that the order quantity is not increased to provide protection against periods of unusually high usage. The order quantity is based upon the average usage, in this example 12,000 pieces per year. Protection is provided by maintaining a safety stock level, through adjustment of the order point. To recap these principles, the following definitions should be reemphasized.

**Active Stock—Order Quantity.** The active stock portion of the inventory is that portion which is created for the purpose of satisfying the *expected* requirements of material. It is directly related to the order quantity. The order quantity is simply the expected annual requirements, divided by that number of orders which has proved to be most economical based upon the cost of ordering and carrying inventory (see Fig. 13-11).

**Safety Stock—Order Point.** The safety stock portion of the inventory is that portion which is created to take care of above-average or unex-

pected demands on the inventory. It is directly related to the order point. The amount of stock is determined not on ordering-carrying cost considerations, but on the need for protection against stockoutages for each stock item under consideration. Some items will need more safety stock than others, depending upon the amount of deviation that has been experienced between the forecasted material usage and the actual material usage for any given time period, plus the reliability of the suppliers' deliveries, and of the order lead time (the longer the lead time, the more uncertain is the forecast of sales, and the resultant material requirements). (Those items whose future activity can be forecasted with 100 per cent accuracy will require, theoretically, no safety stock.)

#### Protection Function Alone

An example of a stock item which is composed almost entirely of safety stock would be a maintenance spare part whose unit cost was high and rate of usage low. Figure 13-7 shows such an item, whose activity is only

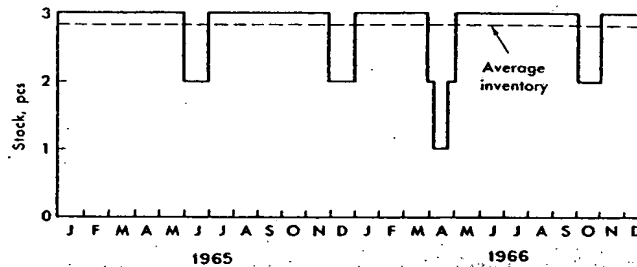


FIG. 13-7. Irregular stock flow and replenishment chart.

two or three units a year. The inventory level, on the average, is close to three units, or \$300. Assuming any reasonable cost factors for ordering and carrying this inventory,<sup>4</sup> it would be cheaper to carry no inventory, and order each piece as needed. The three units of stock, then, are carried purely for reasons of protection, rather than economies of ordering or carrying stock. Such an item, although used infrequently, may be extremely valuable if it can prevent the breakdown of some piece of vital equipment. Since it takes a month to procure the item, there is the possibility that two units could be needed in a given month, and very rarely three units. The stock level is therefore set at three units.

The foregoing discussion of "active stock" and "safety stock" does not suggest that a given stock item should be physically separated into two parts. It does suggest that the two functions be recognized and analyzed

<sup>4</sup> See discussion of cost factors under "Factors Influencing Inventory," page 13-5.

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separately, so that an inventory policy may be established which will result in the desired amount of protection against stockoutages at the minimum over-all cost.

### ORDER QUANTITY

#### Maximum-minimum System

Before discussing the concept of "economical order quantity," the maximum-minimum system will be considered. This system combines order quantity and order point and operates as follows: Two arbitrary levels of stock are selected, usually expressed in weeks' or months' supply. The stock controllers are instructed to order their stock in such a manner that it stays within the two specified limits. For example, in Fig. 13-8,

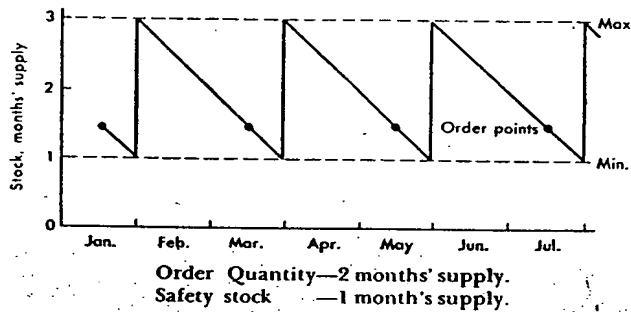


FIG. 13-8. Maximum-minimum system.

the following conditions have been assumed: maximum stock level, three months' supply; minimum stock level, one month's supply; supplier lead time, one-half month. To maintain his stock above the one-month minimum, the stock controller must reorder before his stock drops below  $1\frac{1}{2}$  months. He expects the order to arrive when the stock is about one month's supply. He must not order in quantities in excess of two months' supply, because this order, plus his minimum stock, must not exceed three months. This system has some advantages as well as some serious disadvantages.

#### Advantages:

1. It prevents excessive build-up of stock on any given item, because of the three-month maximum.
2. It provides a level of protection against unusual demands on the stock, because of the minimum level.

3. The system is easy to explain to operating personnel.
4. Actual performance can easily be checked against the standard.

Disadvantages:

1. It is not necessarily the most economical system, when costs of processing orders and carrying inventory are considered.
2. The minimum stock may give either too much or too little protection for specific stock items.
3. The system tends to be *too automatic*. Repeatedly ordering to raise a minimum stock to the maximum level can lead to an overstocked position automatically; frequently design changes are given too little consideration in the strict compliance with maximum-minimum principles. An appropriate check against future requirements must be made to avoid reorders based only upon past experience.
4. Quantity discounts may be lost because of order-quantity restrictions.
5. It does not specifically define either the order point or the order quantity.

To illustrate point 5, under "disadvantages," it can be seen in Fig. 13-9 that there can be an almost limitless combination of order points

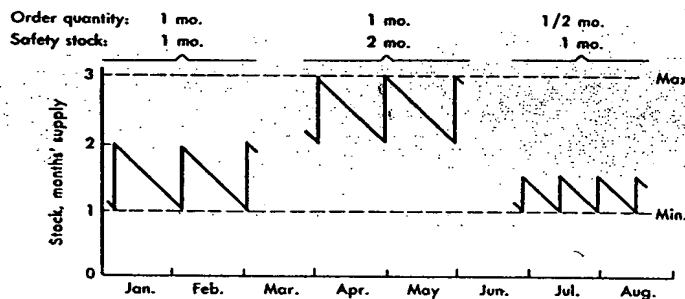


FIG. 13-9. Maximum-minimum system.

and order quantities which will maintain the stock within the prescribed limits. Figure 13-8 shows one method, where the stock fluctuates the full range between maximum and minimum. Each of the patterns in Fig. 13-9 also satisfies the requirements, but each was a different pattern of order quantity or order point. There is no assurance that the maximum-minimum restrictions will be interpreted the same way by each stock controller, so that a nonuniform system among stock controllers may result. These objections may be overcome by calculating the safety stock and order quantity on a more scientific basis.

If any systematic means, other than pure intuition on the part of the stock controller, is to be used to determine the order quantity, there must first be some method for estimating the expected future usage of each stock item. In many cases, the estimates, or forecasts, may be in error by as much as 25 to 50 per cent, but this is not necessarily a deterrent to ordering in large quantities, provided the following is true:

1. The item is not likely to become obsolete in the near future (the product line is not being redesigned).
2. The item does not have a limited shelf life.
3. Large quantities do not present an unreasonable storage problem.

Assuming, then, that there is available an estimate of the annual activity of each item, the stock controller must make a decision as to the quantity of each item he should order. (This assumes that he is not operating under a system which limits his stock on every item to two or three months' supply.) The decision which he makes as to his order quantity will, in turn, reflect itself in his inventory level. Figure 13-9 illustrates the variety of methods that may be employed to reorder his annual requirements.

In this example, the only restrictions on order quantity are a one month's supply minimum, twelve months' supply maximum. Obviously, some practical limit must be imposed at either extreme. Ordering more frequently than once a month per item will usually result in excessive costs of paperwork, material checking, etc. Ordering more than a year's supply will frequently exceed the limits of predictable future activity.

The item shown in Fig. 13-10 has an estimated annual activity of 1,200 pieces and costs \$1 per piece. If the entire year's supply were purchased in one order, the active<sup>5</sup> stock would fluctuate from a maximum of \$1,200 to a minimum of zero, with an average active stock of about \$600. Alternate methods of ordering this same item may be chosen. If the order frequency is increased, the resulting active stock is reduced. If ordered monthly, in this example, the average active stock would be reduced to about \$50. In general, this rule applies:

$$\text{Average active stock} = \frac{1}{2} \text{ order quantity}$$

In this example, assume an ordering cost of \$3 per order, a carrying cost of 10 per cent per year.

The illustration, which shows five different order quantities, is given to emphasize the point that there are many ways in which the annual requirements of a given item may be ordered. In this case, the resulting

<sup>5</sup> That portion of inventory created by order quantity.

inventory could vary from \$50 to \$600. Given the costs associated with each alternative, however, it is possible to select the most economical method. Without such an analysis, the stock controller must rely on intuition or experience to make these decisions.

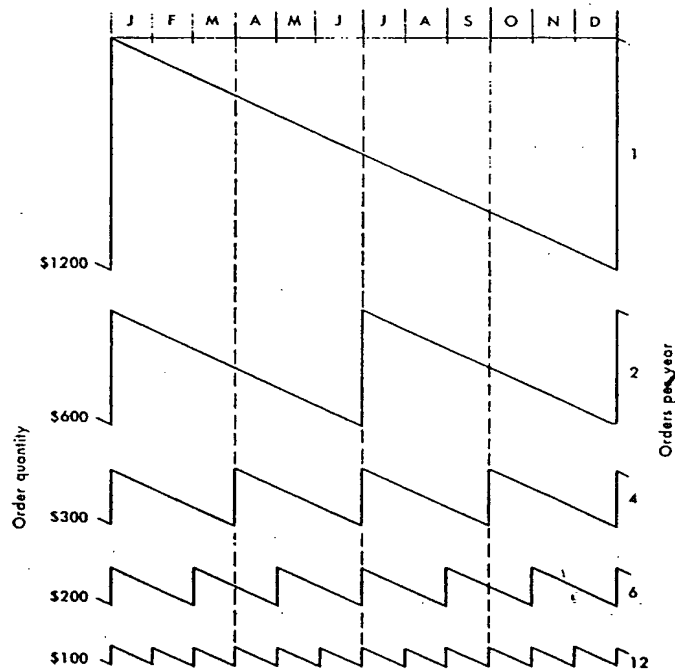


FIG. 13-10. Maximum-minimum system.

Figure 13-11 illustrates that the costs associated with carrying inventory vary inversely with the number of orders. (These costs are reduced from \$60 down to \$5, as the number of orders is increased from one to twelve.)

Figure 13-11 also shows that the ordering costs increase directly with the number of orders (going from \$3 up to \$36, in this case). Since the total cost is made up of both the ordering and carrying cost, it is this cost which must be minimized, rather than either of the other two costs. In this case, the total cost appears to reach a minimum point somewhere between two and six orders per year. If the costs are plotted as curves (Fig. 13-11), it can be demonstrated that the point of minimum total cost is at the point of intersection of the ordering and carrying cost curves.

This will always be true, regardless of the cost factors involved. Other items, having different annual activities, could be analyzed in a similar

manner, but  
the following

$A$  = annual

$C$  = carrying

$K$  = orderi

Number of orders	Order size	Average inventory	Inventory carrying cost	Ordering cost	Total cost
1	\$1,200	\$600	\$60	\$ 3	\$63
2	600	300	30	6	36
4	300	150	15	12	27
6	200	100	10	18	28
12	100	50	5	36	41

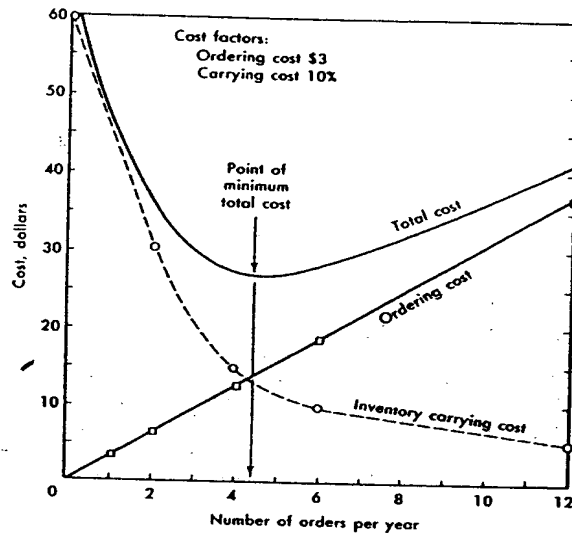


FIG. 13-11. Economical order quantity.

manner, but this is impractical for a large number of items. Therefore, the following formula may be employed:

$A$  = annual requirements (\$)       $N$  = number of orders  
 $C$  = carrying cost-variable (% per year)      Average inventory =  $\frac{1}{2} \frac{A}{N}$   
 $K$  = ordering cost-variable (per order)

Annual ordering cost = annual carrying cost

$$KN = C \left[ \frac{1}{2} \frac{A}{N} \right]$$



Solving for  $N$ , which in this case is the most economical number of orders,

$$N = \sqrt{\frac{CA}{2K}}$$

This formula is a general formula, which may be used for any combination of ordering and carrying costs. The specific cost factors which any given company may have developed can be inserted in this formula. In the example shown, these were \$3 and 10 per cent.

$$N = \sqrt{\frac{CA}{2K}} = \sqrt{\frac{0.10A}{2(3)}} = 0.13 \sqrt{A}$$

This formula may be used to construct a table which relates the annual usage of any stock item to its most economical number of orders per year:

\$ Annual usage $A$	No. of orders $N$
60	1
240	2
540	3
960	4
2,160	6
4,860	9
8,640	12

#### Using the Economical Order Quantity<sup>6</sup>

The EOQ table may be made with many increments, or with just a few, as shown. If computations are done longhand, by the stock controller, it is best that the table be simple and easy to use. The following case shows how the table may be used on a sample item:

1. These facts should be available to the stock controller:

- a. Estimated annual usage, in pieces 4,000
- b. Approximate unit cost 50 cents

2. He performs this calculation:

$$\text{Annual usage (pieces)} \times \text{unit cost} = \text{annual usage (\$)}$$

$$4,000 \times 50 \text{ cents} = \$2,000$$

3. He refers to table for correct number of orders per year:

$$A = \$2,000 \quad N = 6 \text{ (approx.)}$$

4. He calculates this order quantity:

$$\frac{\text{Annual activity}}{\text{No. of orders per year}} = \frac{4,000}{6} = 666 \text{ pieces, or } \$333$$

<sup>6</sup> See tables in Section 29 on how you can order scientifically via the Economic Order Value method.

The calculations described here are simple, and no attempt should be made to carry calculations out to the last decimal place, as this will soon discourage use of the table. By reasonable "rounding of his figures," a stock controller should be able to compute his order quantity within 10 to 15 per cent, without resorting to tedious calculations.

#### *Order Quantity Relative to Lead Time*

The situation may arise where the most economical order quantity is small (say, one month's supply), but the lead time is longer than the order quantity coverage (say, three months). In such cases, in order to cover the procurement cycle, it would be necessary to have several orders open at any given time. In such cases, it may be more convenient to consider the use of a "blanket order," with monthly shipments of specified quantities.

### ORDER POINT

The order point is a predetermined signal which will indicate to the stock controller that he should consider the possibility of reordering the stock item in question. It is expressed in units of material as it is stocked and ordered (pounds, pieces, etc.). Whenever an issue from stock causes the coverage of an item (equivalent to stock balance + open orders) to drop below this predetermined point, the item should be investigated.

The order point must be selected at a figure high enough so that the stock will be sufficient to satisfy the maximum number of expected demands upon the stock during the period when the replacement stock is on order. In brief:

Order point = maximum expected usage during lead time

There are two problems which are inherent in the selection of the proper order point.<sup>7</sup>

1. The lead time cannot always be accurately determined.
2. The usage during the lead time cannot always be accurately forecasted.

In those exceptional cases where both the usage of material and the lead time are absolutely predictable, the order point is simply stated:

Order point = known requirements during lead time

In Fig. 13-12, the lead time is two months, and the known requirements are 100 pieces per month. The order point is therefore 200 pieces.

<sup>7</sup> One exception is in the case of a local supplier who can deliver on a moment's notice. The order point here might be simply zero stock.

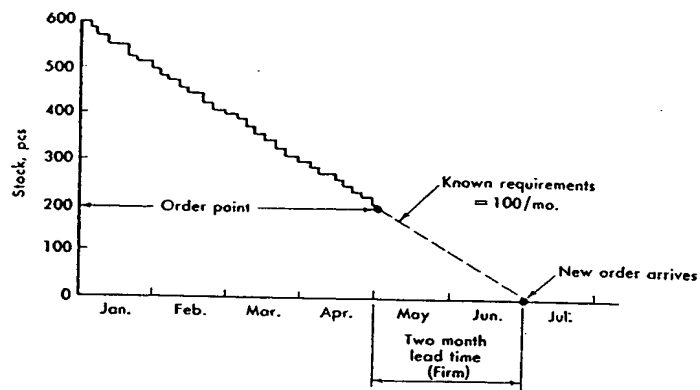


FIG. 13-12. Chart of order-point system.

In the more typical situation, however, the material usage can only be estimated, and the lead time is subject to variations. In the previous example, either a late delivery or a period of above-average activity would have caused the stock to reach zero before the new stock arrived. Therefore, when material usage rates and/or lead times are based on estimates rather than firm figures, it is expedient to make an upward adjustment of the order point. This is done through introduction of safety stock. The order point now becomes:

$$\text{Order point} = \text{expected lead time usage} + \text{safety stock}$$

In the example given previously, there was no safety factor to absorb unexpected usage or delayed delivery. In Fig. 13-13, the order point has

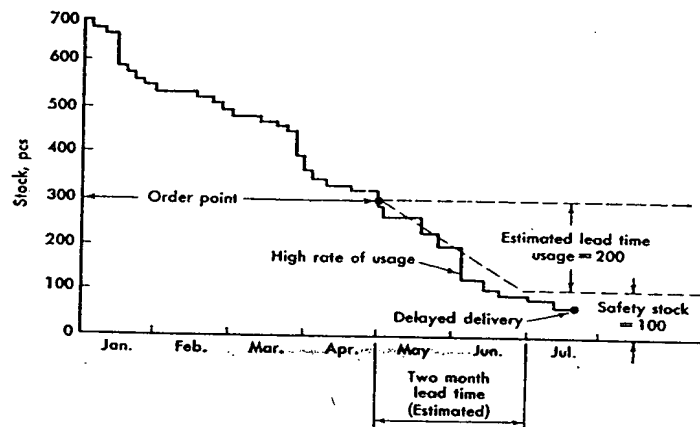


FIG. 13-13. Chart of order-point system with safety stock provision.

been raised 100 pieces, providing a buffer stock which can absorb these unexpected demands.

#### *Calculation of Safety Stock*

The amount of safety stock necessary to provide adequate protection will vary from item to item. This is true because forecasts can be made with greater accuracy for some items than for others. Lacking accurate forecasts, the stock controller may examine past stock records in order to help him estimate the maximum rate of usage which could be expected to occur during the reorder period. He might also determine the longest recorded delivery time for the item. Judgment should be used, however, so that nontypical, nonrecurring situations will be excluded. Based on such an analysis, the stock controller may decide upon an amount of safety stock sufficient to cover those occurrences which are in excess of the average usage and/or lead time.

Lengthy analyses, which may be valuable for an occasional stock item which is critical or of high dollar value, are usually impractical to apply to all items. The problem may be resolved by the use of an across-the-board safety stock. Such a generalized safety stock is often defined as "one month's supply." It means that the stock controller will order in such a manner that he will always have at least one month's supply of stock on hand, if conditions remain normal. If not, he will have the extra month's supply of stock to carry him over the period of high usage or delayed delivery.

Where safety stock is expressed in months' supply, it is convenient to use this formula:

$$[\text{Safety stock (months)} + \text{lead time (months)}] \times [\text{average monthly usage (units)}] = \text{order point (units)}$$

Following are three items, with different activities and lead times. In each case, assume a safety stock of one month's supply.

Item No.	Monthly usage, pieces	Lead time, months	Safety stock, months
1	100	1.0	1.0
2	500	2.0	1.0
3	20	2.5	1.0

The order points are computed as follows:

$$\begin{array}{lcl}
 (\text{Safety stock} + \text{lead time}) (\text{average month's usage}) & = & \text{order point} \\
 \text{Item 1 } (1.0 + 1.0) \text{ 100} & = & 200 \text{ pieces} \\
 \text{Item 2 } (1.0 + 2.0) \text{ 500} & = & 1,500 \text{ pieces} \\
 \text{Item 3 } (1.0 + 2.5) \text{ 20} & = & 70 \text{ pieces}
 \end{array}$$

*Relationship of Safety Stock, Stockout Rate, and Stockout Cost*

The distinction has been made between active stock and safety stock to point out that the active stock is created for purposes of economies of ordering, whereas the safety stock is added only to cover above-average usage or delayed deliveries. Some industries refer to this protection as "contingency stock." The costs associated with carrying this part of the inventory are therefore high. Any effort to reduce the safety stock, however, will automatically increase the number of stockoutages. It is theoretically possible to compare the cost of a stockout with the cost of carrying safety stock, and thereby arrive at the most economical level of safety stock, considering both costs. However, the cost of a stockout for a given item could vary greatly from one occurrence to the next, depending upon its effect upon the production line each time. The actual costs incurred in such a situation are largely a matter of opinion. This makes it impossible, in most cases, to determine a realistic stockout cost.

Although the actual costs associated with a stockout may be difficult to determine, it is not difficult to tabulate the rate of stockouts and the average duration of the stockouts. These data can be analyzed to determine if the inventory is "in control." Past history, combined with management judgment, should be enough to determine an acceptable maximum stockout rate, beyond which the manufacturer of the product will suffer. This rate might be different for various commodities. A reporting system should be set up so that actual stockouts per month may be compared with the standard. When stockouts become excessive, the safety stock must be increased until the stockouts are reduced to an acceptable level. It should be noted that an item which never goes on shortage should be reviewed critically. It may be that the stock is excessively high, and that an occasional stockout would cost less than carrying excess inventory year after year.

*Refinements in Safety Stock—Order-point Calculations*

Although it is true that the stockout rate will decrease as the safety stock is increased, it is not a simple matter to determine the exact amount of safety stock necessary to keep the stockouts within a given limit. Such rule-of-thumb methods as a month's supply across-the-board safety stock may be successful in keeping the over-all rate down, but individual items may go on shortage much more frequently than desired for good control. This emphasizes the need for an adjustable safety stock, tailored to the needs of each individual stock item.

Some of the factors which influence the level of safety stock are intuitively considered by most stock controllers. An examination of many stock

ledgers will show different levels of safety stock from item to item in line with these factors.

The following list suggests some of the factors which should be considered in determining the correct level of safety stock.

Factors which suggest low safety stock

1. High permissible stockout rate (one stockout every year, per item)
2. Short delivery time (one-half month)
3. Large-order quantities (twelve months' supply per order)
4. Stable item
  - a. Issued many times per month
  - b. Predictable usage
  - c. Used in a variety of products

Factors which suggest high safety stock

1. Low permissible stockout rate (one stockout every ten years, per item)
2. Long delivery time (six months)
3. Small-order quantities (one month's supply per order)
4. Erratic item
  - a. Issued only once a month or less
  - b. Unpredictable usage
  - c. Used in only one or two products

Applying these factors to an inventory which contained a wide variety of items could result in a range of safety stocks from a one-half month supply for short-lead-time, fast-moving items to as much as a six-month supply for relatively slow movers with long lead times. The relation between the factors and the resulting safety stock can best be determined by extensive statistical analysis. Many of the larger companies have successfully made such studies and, by application of the laws of probability, devised control tables which give the correct safety stock or order point for a wide variety of conditions.

## QUANTITY DISCOUNTS

If an item of raw material or a purchased part is being purchased for special application, rather than for inventory, there arises the problem of whether or not to increase the requested quantity to take advantage of a quantity discount. An item for which future activity seems unlikely should be purchased in the quantity requested, since there is a known requirement for that amount only. The person, frequently the design engineer, who specified the item in question could probably give the buyer some idea of the likelihood for future use; the buyer might consider increasing the order quantity, provided that the resulting price reduction was justified, in the light of existing inventory policies. Two factors which must be considered are the potential reduction

in material cost versus the increase in inventory. Because the inherent risk of obsolescence is great, in such cases, it is advisable that the potential price reduction be significant compared to the increased inventory investment. The ratio of the former to the latter may be expressed as a percentage. Frequently an arbitrary minimum percentage is decided upon, below which the discount will not be taken. For example, it may be decided that no item of a certain commodity will be ordered in excess of actual requirements unless a 30 per cent return on investment can be realized.

## EXAMPLE:

$S$ = requested order quantity (units)	= 200 pieces
$S_d$ = minimum order quantity for discount (units)	= 1,000 pieces
$P$ = unit price at requested order quantity	= \$1.00
$P_d$ = unit price at discount quantity	= \$0.80
$Q$ = requested order quantity in \$ = $S \times P = 200 \times \$1$	= \$200
$Q_d$ = discount order quantity in \$ = $S_d \times P_d = 1,000 \times \$0.80$	= \$800
Material price reduction = $S_d(P - P_d) = 1,000(\$1 - \$0.80)$	= \$200
Increase in inventory value = $Q_d - Q = \$800 - \$200$	= \$600

$X'$  = per cent return on additional investment

$$= 100 \times S_d \frac{(P - P_d)}{Q_d - Q} = 100 \times \frac{200}{600} = 33\frac{1}{3} \text{ per cent}$$

Since the actual per cent return exceeds the 30 per cent minimum, the discount would be taken on 1,000 pieces.

A more scientific method is to tabulate a "before and after" price-break chart. If the savings exceed the cost of carrying the additional inventory,

Total dollar amount after quantity discount	Total dollar amount before quantity discount						
	10	50	100	150	200	500	1,000
20	0.60						
50	2.40						
100	5.40	3.00					
200	11.40	9.00	6.00	3.00			
300		15.00	12.00	9.00	6.00		
400			18.00	15.00	12.00		
500				21.00	18.00		
700					30.00	12.00	
900						24.00	
1,200						42.00	12.00
1,500						60.00	30.00

FIG. 13-14. Added inventory costs.

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the discount is taken. If the savings are less than the cost of carrying the additional inventory, the lower quantity, at the higher unit price, would be more economical.

Assuming a (variable) carrying cost of 12 per cent, and basing the average inventory on one-half the (additional) order quantity, a sample price-break chart would look like Fig. 13-14. The horizontal column headings are the total dollar amounts for an item in the required quantity. The vertical headings are the total dollar amounts to be spent if the quantity unit-price discount is to be taken. As an example, take the column indicating a "before" cost of \$100. If an expenditure of \$200 is required to take the discount, then a carrying cost of \$6 is incurred. (By going from \$100 to \$200 an additional order amount of \$100 is committed. Average inventory would be  $\frac{1}{2} \times \$100 = \$50$ . Carrying costs of 12 per cent produce an inventory cost of \$6.)

#### EXAMPLE 1:

The economic order quantity for an item is 110 pieces, at a cost of 90 cents each, for a total of \$99. The unit cost reduces to 80 cents each in a quantity of 250 and up, for a total of \$200. Annual usage is 150 pieces. The chart shows that an inventory cost of \$6 is incurred by going to the larger quantity. The usage of 150 pieces produces savings of  $150 \times \$0.10 = \$15$  per year. There is a net gain of \$9 on an annual basis, and the discount should be taken.

#### EXAMPLE 2:

The economic order quantity for an item is 40 pieces at \$5 each, for a total of \$200. A reduction of the unit price to \$4.70 each is available in a minimum quantity of 150 pieces, requiring an investment of \$705. Annual usage of this item is 80 pieces. Reference to the chart shows added-inventory carrying costs of \$30. Annual savings ( $80 \times \$0.30 = \$24$ ) are less than the cost of carrying the additional inventory. The discount should not be taken.

## INVENTORY CONTROL AND EXPEDITING

### Effect of Expediting on Inventory

A study of inventory-control techniques would not be complete without a discussion of the effect of expediting on inventories. The expediting of delivery of procured materials has a major effect upon inventory levels since it influences directly the lead time or procurement-cycle time.

As indicated in previous discussions, a predetermined inventory level is usually maintained through proper consideration of the procurement-cycle time when placing orders for a new supply of material. This consideration includes a knowledge of the supplier's manufacturing cycle,





principles, practices, makes use of standard al and economical.

which illustrate in mechanized purchase use of TDP and EDP actions of the system.

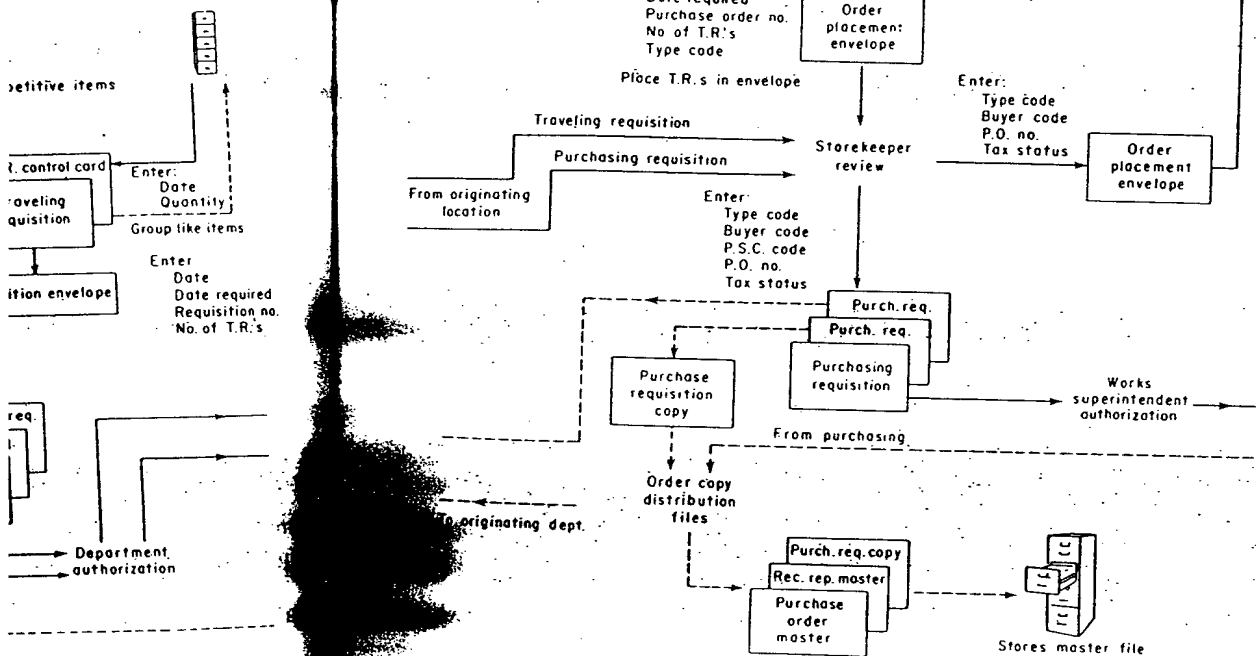


FIG. 14-7b. Ordering by traveling requisitions in a mechanized purchase order writing system—employing manual, Flexowriter, key-punch, tape-to-card, and computer processes.

base order writing system—nd computer processes.

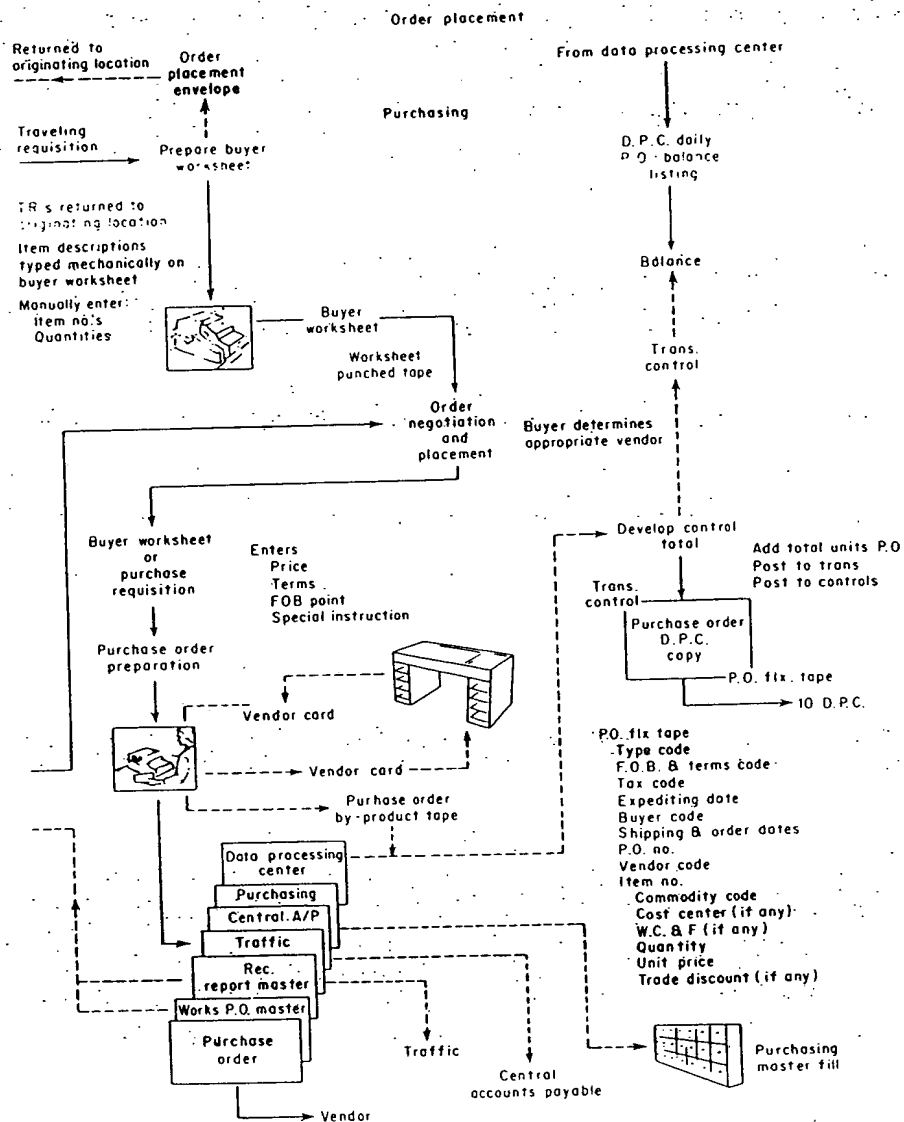
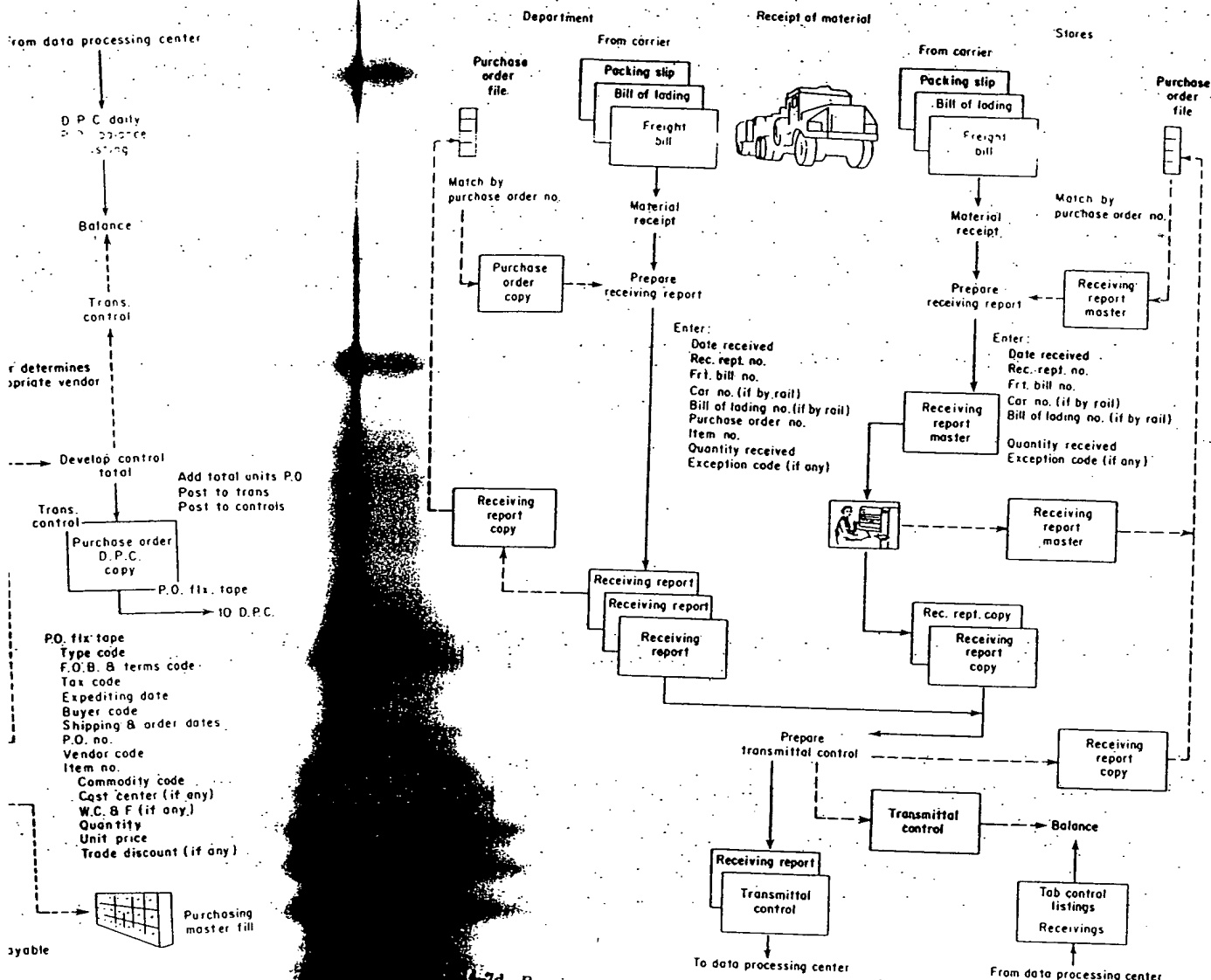


FIG. 14-7c. Buyer processing and purchase document preparation in a mechanized purchase order writing system—employing manual, Flexowriter, key-punch, tape-to-card, and computer processes.



1. Preparation in a mechanized purchase order writing system employing manual, Flexowriter, key-punch, tape-to-card, and computer processing.

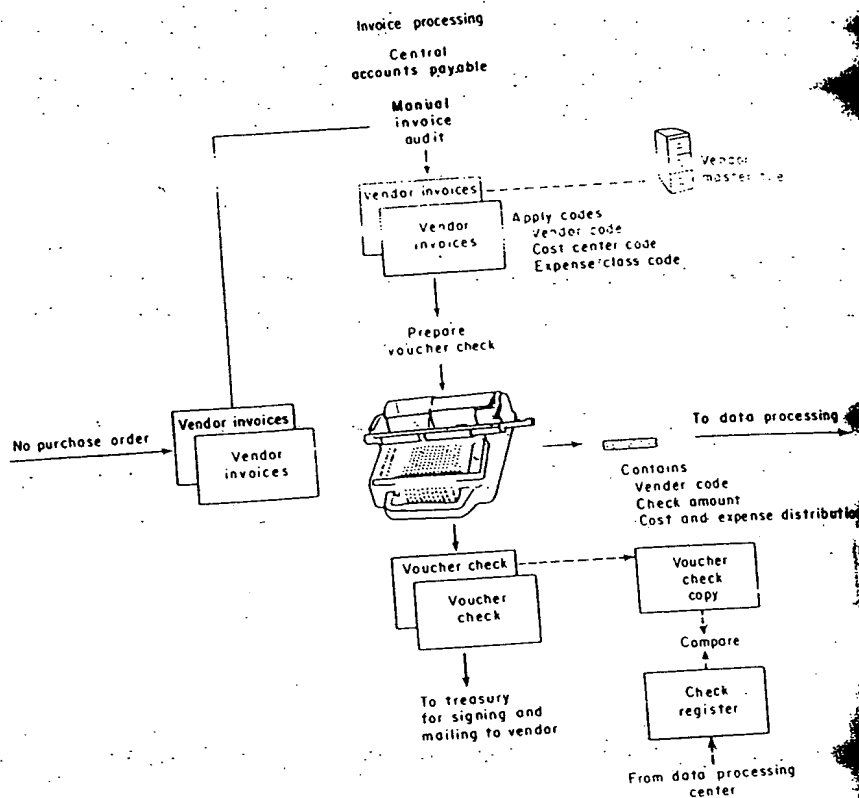


FIG. 14-7e. Manual invoice processing in a mechanized purchase order writing system employing manual, Flexowriter, key-punch, tape-to-card, and computer processes.

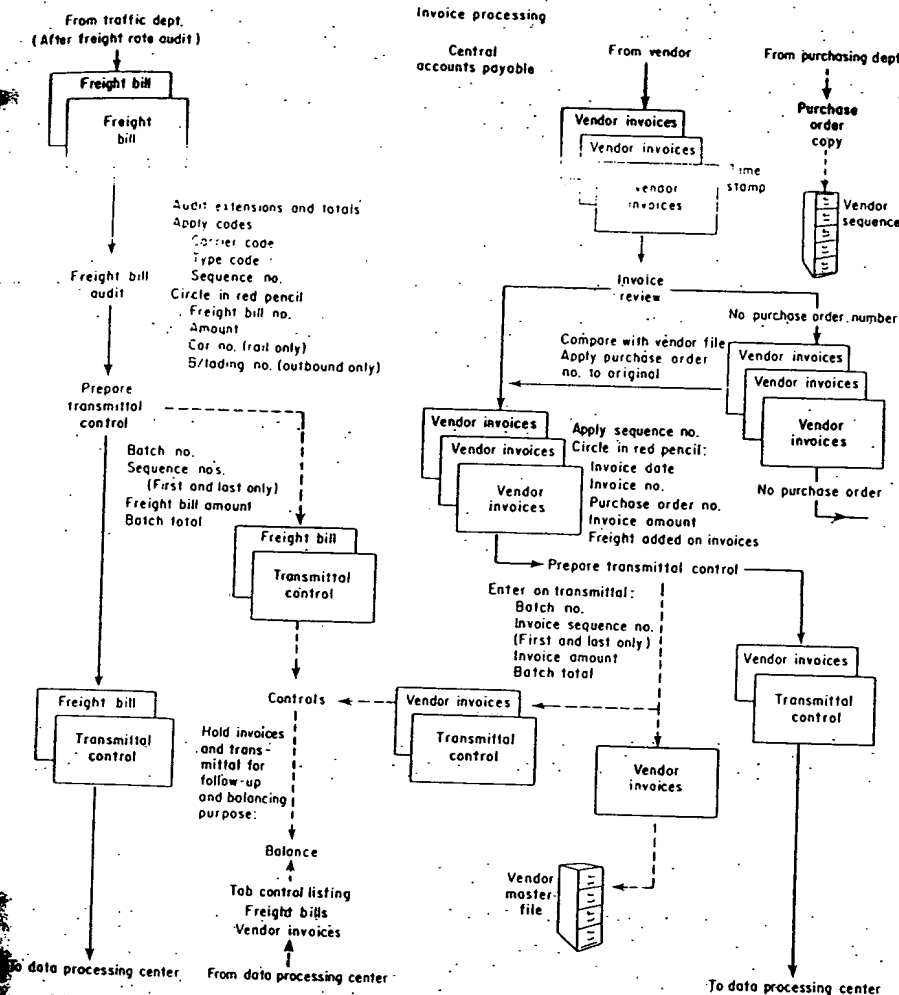


FIG. 14-7f. Preparation of freight and invoice computer data in a mechanized purchase order writing system—employing manual, Flexowriter, key-punch, tape-to-card, and computer processes.

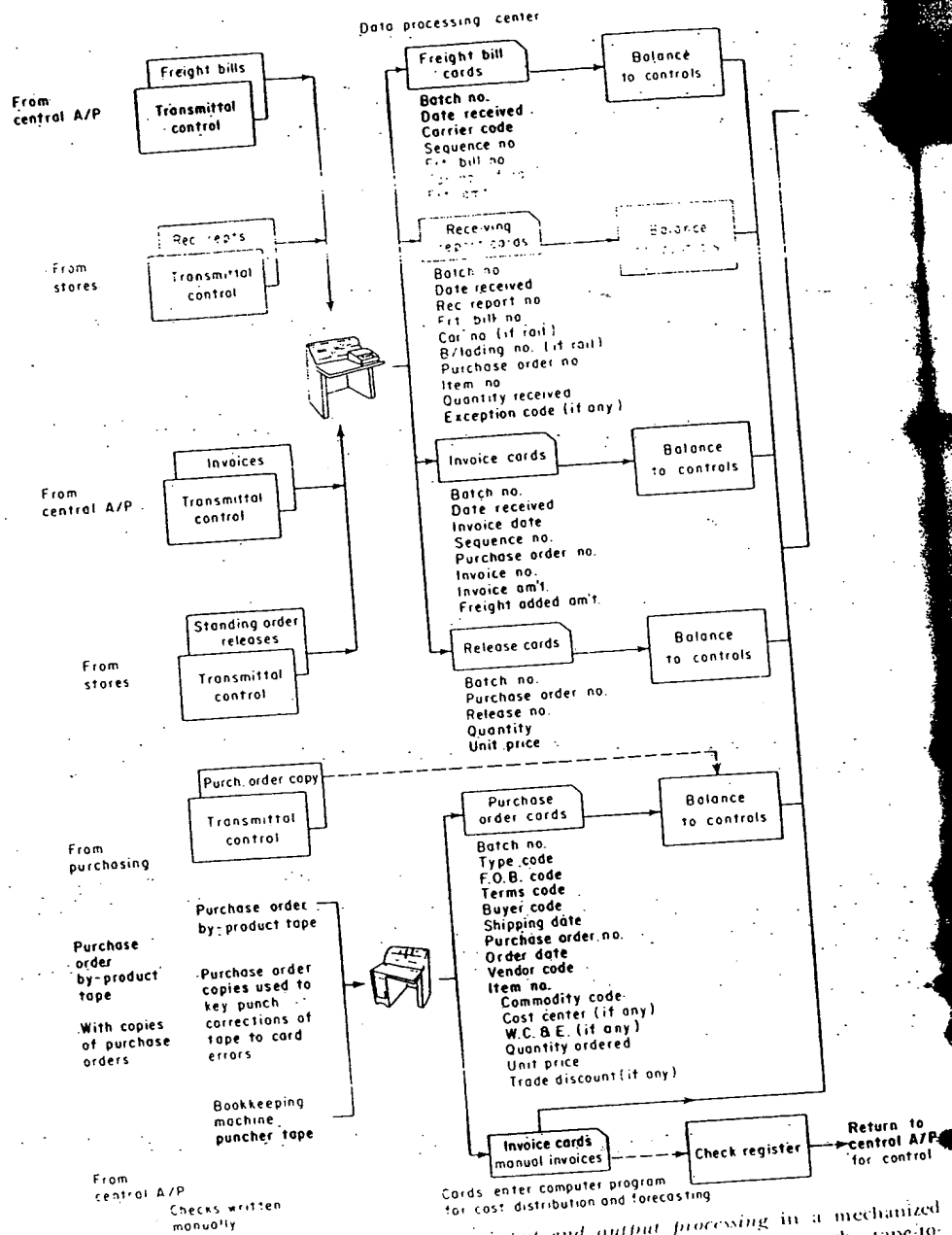


FIG. 14-7g. Accounts payable computer input and output processing in a mechanized purchase order writing system—employing manual, Flexowriter, key-punch, tape-to-card, and computer processes.

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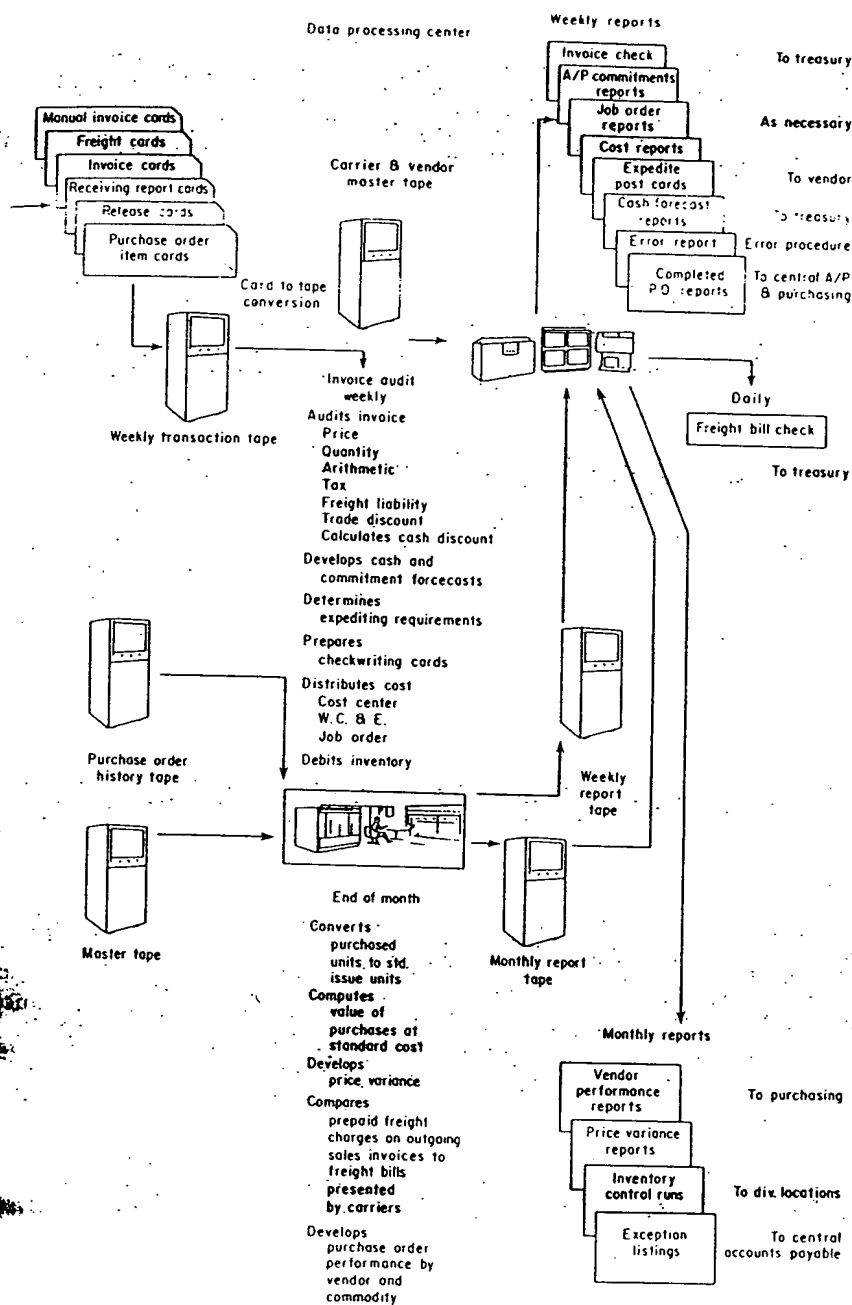


FIG. 14-7h. Accounts payable computer processing and purchasing statistics developments in a mechanized purchase order writing system—employing manual, Flexowriter, key-punch, tape-to-card, and computer processes.



This system is dependent on a uniform commodity code and uniform specifications as its bases. Although the mechanical order writing system uses Flexowriters and combines principles of both EDP and IDP, it is flexibly designed and would enable a transition to buying highly repetitive items by computer to be made with a minimum of cost and effort if this course of action is desired by the company.

#### EXPEDITING, VENDOR PERFORMANCE, EOQ

The complete EDP purchasing cycle, as noted, provides all of the basic data to initiate expediting, vendor performance, economic order quantities (EOQ), and other types of management tools. There exist so many varied approaches and possibilities for obtaining controls and information relative to the above that it would be impractical to discuss details. Again the development and design of these programs will depend on the company EDP objectives, and each tool must be tailored to fit specific needs of each company.

#### INVENTORY MANAGEMENT

Since the theory of inventory control and reordering is so closely allied to purchasing functions, a detailed discussion will follow on this particular subject. The balance of this section will deal exclusively with the principles, concept, and theory which relate to inventory management through EDP.

1. There is a description in some detail of how a mathematical model is constructed.
2. The importance of using operating or functional people in planning, particularly that of the input, is underlined.
3. The need for absolute accuracy of data is stressed, as is the importance of training people to properly provide it and appreciate its importance.
4. The principle of management by exception using control limits is outlined, and it is pointed out that these limits should be consistent with the cost of adhering to them.
5. The point is made that extreme precision of the model, obtained at high cost, is unwarranted when the forecast of demand is itself incapable of such precision.
6. There is an exhibit of a typical "exception report" and a discussion of its use.
7. Performance reporting is discussed.
8. The importance of coupling human judgment and power of decision

tion regulations and applies to all Federal agencies. Copies of these regulations have been furnished, upon request, to various states through their national associations.

Some state statutes specify that the purchasing agency shall arrange for the transfer to or between agencies of personal property which is surplus with one but which may be needed by others. Some states are carrying on an effective program for the utilization of state-generated surplus property, with the result that many purchases of new and additional property are avoided, with resultant savings in tax funds.

#### *Types of Contracts*

In public purchasing the fixed-price, definite-quantity contract is usually preferred over other types. There are, however, circumstances where other-type contracts are more advantageous to both parties. Cost-type contracts, fixed-price incentive contracts, cost-plus-incentive-fee, and like varieties of contracts with special features are not discussed since they are used primarily in connection with military-type production programs. The use of "cost plus a percentage of cost" contracts is specifically forbidden in General Services Administration regulations of the Federal government. The following categories and specific types of contracts represent those in general use in public purchasing departments.

*Indefinite-quantity or open-end contracts* are widely used in government purchasing because of two major advantages. First, they permit contracting for total requirements of all departments or agencies on a long-term basis and provide minimum prices in those situations where exact-quantity requirements cannot be determined. Second, they have a tendency to minimize the number of items being ordered since the process of consolidating requirements results in a definite standardization of the best types or qualities of products. The relative simplicity of purchase actions under this type of contract, and the elimination of duplicative supply contracts by the various departments or agencies, results in substantial economies and many administrative advantages.

The General Services Administration uses *indefinite-quantity or term contracts* in the procurement of common-use items under its Federal Supply Schedules. Under such schedules all Federal agencies have the benefit of prices based on total quantity requirements even though the orders issued by each for direct delivery are relatively small. Federal Supply Schedules cover approximately 50,000 items or categories of items, including office supplies, office furnishings and equipment, and tires and tubes.

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"supply" or a "requirements" contract. Whatever the name used, it represents an agreement whereby the supplier contracts to furnish the total requirements of the government for a specified item over a stated period of time, for delivery as ordered. The price may be a "fixed price," or it may be subject to fluctuation of an established pricing medium such as posted prices for gasoline.

Counties, cities, and public institutions also use the supply contract or agreement to cover such items as chemicals, lamps, fuels, paper products, sand and gravel, laundry supplies, and photographic and duplicating supplies.

Long-term contracts are used in practically all public purchasing activities as a means of securing quantity discounts, and because of the savings effected by only advertising for bids once during the period covered by the contract. Such contracts may require performance bonds and do not ordinarily extend beyond one year.

*Definite-quantity contracts*, which may also be *term contracts*, are used where requirements are known or can be estimated with reasonable accuracy. They may provide for "one-time" delivery or "scheduled" delivery, and the price may be a *fixed price* with or without provisions for an escalation clause. Definite-quantity contracts make it possible to take advantage of quantity prices prevailing at the time of the contract while spreading delivery over future periods. Other advantages are (1) the vendor can use such contracts as a backlog to keep his plant busy and as a result will usually give the buyer a better price; (2) the number of purchase orders and related forms are reduced; (3) the cost of advertising on formal contracts is limited to the initial contract for the period covered; (4) many small orders are eliminated; and (5) price advantage is obtained while inventory carrying costs are maintained at a low level.

*Price-agreement contracts* are those where certain discounts from list prices are incorporated in the agreement. Such discounts may be part of any contract that covers the procurement of either cataloged items or standard production items, where the price is reflected by published price lists.

*Service contracts* are normally used to cover requirements for services such as window washing, utilities, laundry, insect extermination, guard service, truck rental, and repair services. They may be formally advertised or negotiated contracts depending upon considerations such as the number of sources of supply and the dollar value of the contract.

Samples of general provisions prescribed for use in the Federal government and also by the state of Connecticut will be found at the end of this section.

### *Escalation Clauses*

In long-term contracts, where the possibility of increased labor or material costs may affect the supplier's willingness to set an acceptable price, escalator or price revision clauses may be used. Such "flexible" contracts apply to both definite and indefinite-quantity agreements.

Escalation or price-revision clauses in bids are not usually desirable. They were in general use, however, during World War II and the Korean conflict. Under certain circumstances they serve a useful purpose in protecting both the government and the supplier from either a rising or falling market. In some cases suppliers must quote an obviously excessive price to provide for the contingencies of rising costs of labor and materials. Under these conditions it may prove advantageous for the bid or contract to provide for escalation clauses, if such clauses contain a maximum price, percentage ceiling, or are related to a public price information service such as the *Wall Street Journal* or *Chicago Journal of Commerce*. In some instances where no bids are received because of unstable market conditions it may be necessary to readvertise and include an escalation clause.

Where escalation clauses are used, they should be included in the invitation to bid and contain a maximum price or percentage ceiling. Such maximum should be based on the best estimate which can be made of expected increases in labor or raw materials, plus any other pertinent factors. Current Armed Services' and certain civilian agency regulations use a 10 per cent figure. However, the circumstances of each individual case should determine the maximum percentage of increase.

Regulations should include *specific* instructions concerning the use of escalation clauses, and should provide for approval by the head of the purchasing department, prior to use.

### *Inspection of Deliveries*

The inspection of materials and equipment prior to their acceptance is one of the most important responsibilities in the entire procurement operation. This is especially true in public purchasing as the act of acceptance by inspection may be irrevocable. Once the goods are accepted, litigation is sometimes required to effect adjustment on items found to be of inferior quality.

Inspection is defined, for use herein, as the method or process of determining that materials, equipment, and services supplied by a contractor or vendor meet the quality requirements stipulated in the contract or other purchase documents.

# CONTINGENT CLAIMS CONTRACTING FOR PURCHASING DECISIONS IN INVENTORY MANAGEMENT

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Option pricing is a common and important practice in the financial community, and has become a fundamental theoretical construct in financial economics. The theory is quite rich and has potential uses in many other problem domains. This paper develops a variant of the theory as applied to inventory planning. In particular, we consider a risk management approach that uses negotiated option contracts for hedging against price and quantity uncertainty in inventory procurement. We derive conditions for the inclusion of options in inventory control as a function both of managerial attitudes toward risk and of the correlation between price and demand.

Contingent claims contracts provide purchasing agents with the right to buy a specified amount of goods at a fixed price from suppliers at a future time. Such contracts serve as effective risk management devices that allow agents to adapt to changing conditions, prices or demands. By negotiating such contracts, the agent can delay commitments until some, or all, uncertainty has been resolved.

To see how contingent contracts can reduce risk, consider a manager who can place only a single order before a random demand occurs. If demand exceeds supply, new orders cannot be expedited since the supplier carries no finished inventory. Option contracts provide a mechanism for enticing the supplier to build finished inventories, even though the purchasing agent makes no firm commitments. The price of the contract should be set sufficiently high to compensate the supplier in the event no second order is placed. In exchange for this price, the purchasing agent transfers the risk associated with demand uncertainty to the supplier. This transfer may be advantageous to both parties, especially if the supplier is large and is more able to absorb the risk.

This article considers the design of an optimal mesh of contingent claims (options) with purchasing commitments that will best meet the risk-reward preferences of the decision maker. For example, a purchasing agent may choose to build inventory for future demand, enter an options contract for a fixed price, or wait until demand occurs and purchase at the future spot price. Under specific risk preferences, this article examines conditions under which option contracts serve as superior or complementary strategies to inventory building.

Unlike this paper's scope, the financial economics literature has emphasized the valuation of option contracts. Black and Scholes (1973), for example, derived an option pricing model that was independent of preferences. The simplicity of their model stems from their continuous trading premise, which is equivalent to a complete markets assumption (Bensoussan 1984, Harrison and Pliska 1981). This being the case, the option is redundant and can be unambiguously valued, using arbitrage arguments alone. With discrete trading opportunities and transaction costs, however, option prices will depend on preferences (Rubinstein 1976). In this article, we assume option prices are provided, and our concern is to use these contracts as hedging devices to manage the price and demand risk of inventories. For a review of option theory and pricing, see Merton (1973) and Smith (1976).

## 1. Notation and Assumptions

Let  $V_0$  be the number of units acquired at  $i = 0$ , at a per-unit price of  $S_0$ , and stored for use in period  $i = 1$ . Let  $U_0$  be the number of options purchased at  $i = 0$ . Each contract gives the owner the right to buy one unit of the item at a fixed price  $\pi$  in period  $i = 1$ . The per-unit cost of each contract is  $C_0$ . Order quantities in period  $i = 1$  are determined once prices  $S_1$  and demand  $d_1$  have been resolved. At  $i = 0$ , the future prices and demands can be represented by a known probability distribution. Let  $\tilde{S}_1$  and  $\tilde{d}_1$  represent the random variables characterized at  $i = 0$ .

Let  $F(V_0, U_0)$  be a random variable representing the present value of costs for acquiring  $V_0$  units of inventory and  $U_0$  options, carrying them for one

*Subject classification:* 332 hedging price and quantity uncertainty with options, 362 dynamic program for inventory order quantities, 855 purchasing decisions under quadratic risk aversion.

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period and subsequently taking the appropriate purchasing decisions to ensure future demand  $d_i$  is met. Let  $h_s(V_0)$  and  $h_c(U_0)$  be the present value of the inventory and option carrying charges. The former cost includes storage insurance and cost of money charges, whereas the latter consists primarily of the cost of funds.

The present value is a random variable since future order quantities in period  $i = 1$  are not currently known. These quantities are determined in period 1, once prices and demand have been resolved.

In addition to buying  $V_1$  items in period 1, the purchasing agent either exercises the  $U_0$  options or not. The decision will depend on whether market price  $S_1$  exceeds the exercise price  $\pi$ . The inherent value  $C_1$  of each contract in period 1 is

$$C_1 = \max(0, S_1 - \pi). \quad (1)$$

We shall assume that the market is at least efficient enough that this no-arbitrage-opportunity condition is met (Merton). With this assumption, the decision maker will be indifferent to exercising the options or selling the contracts back to the supplier and purchasing at the spot price. With no loss of generality, we shall assume that the agent will adopt the latter strategy.

With the prior assumptions, the net present value of costs for satisfying future demand is

$$F(V_0, U_0) = S_0 V_0 + h_s(V_0) + C_0 U_0 + h_c(U_0) - \delta \tilde{C}_1 U_0 + \delta \tilde{S}_1 V_1^*, \quad (2)$$

where  $\delta$  is the discount rate and  $V_1^*$  is the optimal number of units that must be purchased in period 1. Since all demand is to be met,

$$V_1^* = d_1 - V_0. \quad (3)$$

At  $i = 0$ ,  $V_1^*$  is random, since  $d_1$  is random.

The decision maker is assumed to base current decisions so as to minimize expected disutility of the net present value of costs incurred in satisfying demand. The optimization problem at  $i = 0$ ,

$$\min_{V_0, U_0} E_{S_1, d_1} \mathcal{L}(F(V_0, U_0; S_1, d_1)), \quad (4)$$

is defined by a disutility (loss) function  $\mathcal{L}(\cdot)$  associated with decisions  $V_0$  and  $U_0$ .

## 2. Inventory Options and Risk Aversion

To illustrate the conditions under which contingent claim contracts provide a mechanism for purchasing agents to reduce risk, consider the case of a risk-averse decision maker with quadratic utility. In this case,

decisions can be based on means and variances alone (Markowitz 1952). Levy and Markowitz (1979) have shown that functions of means and variances provide close approximations to expected utility for important classes of utility functions, including logarithmic, power and exponential functions. Studies by Pulley (1983) have provided further empirical support for mean variance approximations.

For this case, Equation 4 reduces to

$$E \mathcal{L}(F(V_0, U_0; S_1, d_1)) \\ = E(F(V_0, U_0; S_1, d_1)) + \lambda \text{Var}(F(V_0, U_0; S_1, d_1)). \quad (5)$$

In this expression,  $\lambda$  is the risk aversion parameter, often referred to as the price of risk.

Assuming that carrying charges are linear, i.e.,

$$h_s(V_0) = h_s V_0 \text{ and} \\ h_c(U_0) = h_c U_0,$$

we have

$$E(F(V_0, U_0; S_1, d_1)) \\ = (S_0 + h_s - \delta E(\tilde{S}_1))V_0 + (C_0 + h_c - \delta E(\tilde{C}_1))U_0 \\ + \delta_1(E(\tilde{S}_1)E(\tilde{d}_1) + \text{Cov}(\tilde{S}_1, \tilde{d}_1)), \quad (6)$$

and the variance is given by

$$\text{Var}(F(V_0, U_0; S_1, d_1)) \\ = \delta^2 \text{Var}(\tilde{S}_1)V_0^2 + \delta^2 \text{Var}(\tilde{C}_1)U_0^2 \\ + 2\delta^2 \text{Cov}(\tilde{S}_1, \tilde{C}_1)U_0 V_0 \\ - 2\delta^2 \text{Cov}(\tilde{S}_1, \tilde{S}_1 \tilde{d}_1)V_0 \\ - 2\delta^2 \text{Cov}(\tilde{C}_1, \tilde{S}_1 \tilde{d}_1)U_0. \quad (7)$$

Substituting (6) and (7) into (5), and rearranging, we obtain a quadratic expression in the decision variables  $V_0$  and  $U_0$ . Solving the equation yields

$$V_0^* = \frac{a_1 \text{Cov}(\tilde{S}_1, \tilde{C}_1) - a_1 \text{Cov}(\tilde{C}_1)}{[\text{Cov}^2(\tilde{S}_1, \tilde{C}_1) - \text{Var}(\tilde{S}_1)\text{Var}(\tilde{C}_1)]}, \quad (8)$$

and

$$U_0^* = \frac{a_1 \text{Cov}(\tilde{S}_1, \tilde{C}_1) - a_2 \text{Var}(\tilde{S}_1)}{[\text{Cov}^2(\tilde{S}_1, \tilde{C}_1) - \text{Var}(\tilde{S}_1)\text{Var}(\tilde{C}_1)]}, \quad (9)$$

where

$$a_1 = b_1 + \text{Cov}(\tilde{S}_1, \tilde{S}_1 \tilde{d}_1), \\ a_2 = b_2 + \text{Cov}(\tilde{C}_1, \tilde{S}_1 \tilde{d}_1), \\ b_1 = [\delta E(\tilde{S}_1) - (S_0 + h_s)]/2\lambda\delta^2, \text{ and} \\ b_2 = [\delta E(\tilde{C}_1) - (C_0 + h_c)]/2\lambda\delta^2. \quad (10)$$

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In order to analyze the solution, it is useful to define partial moments about the exercise price  $\pi$ . Specifically, let

$$Y_k = \int_{\pi}^{\infty} s_1^k dF(s_1), \quad (11)$$

where  $F(s_1)$  is the cumulative price distribution function. Then

$$E(\tilde{C}_1) = Y_1 - Y_0, \quad (12)$$

$$\text{Var}(\tilde{C}_1) = Y_2 - \pi Y_1 - E(\tilde{C}_1)(\pi + E(\tilde{C}_1)),$$

and

$$\text{Cov}(\tilde{S}_1, \tilde{C}_1) = \text{Var}(\tilde{C}_1) + E(\tilde{C}_1)(\pi + E(\tilde{C}_1) - E(\tilde{S}_1)).$$

To interpret the optimal solution, we shall investigate some special cases.

#### Case 1: No Options Available

In this case the optimal order quantity  $V_0^*$  is obtained by putting  $U_0^* = 0$  in Equation 5, differentiating with respect to  $V_0$ , and solving. The optimal solution is

$$V_0^* = \frac{\delta E(\tilde{S}_1) - (S_0 + h_s) + \text{Cov}(\tilde{S}_1, \tilde{S}_1 \tilde{d}_1) 2\delta^2 \lambda}{2\delta^2 \lambda \text{Var}(\tilde{S}_1)}. \quad (13)$$

If, in addition, demand and price are assumed to be independent, then

$$\text{Cov}(\tilde{S}_1, \tilde{d}_1) = 0 \quad (14)$$

and

$$\text{Cov}(\tilde{S}_1, \tilde{S}_1 \tilde{d}_1) = E(\tilde{d}_1) \text{Var}(\tilde{S}_1). \quad (15)$$

In this case, Equation 13 reduces to

$$V_0^* = E(\tilde{d}_1) + \frac{\delta E(\tilde{S}_1) - (S_0 + h_s)}{2\lambda \delta^2 \text{Var}(\tilde{S}_1)}. \quad (16)$$

Thus, given that the expected discounted price  $\delta E(\tilde{S}_1)$  exceeds the current price-plus-carrying-charges, the quantity ordered will decrease with price uncertainty and risk aversion.

#### Case 2: No Inventory Building

In this case,  $V_0^* = 0$  and the optimal number of option contracts to purchase is

$$U_0^* = \frac{\delta E(\tilde{C}_1) - (C_0 + h_c) + 2\lambda \delta^2 \text{Cov}(\tilde{C}_1, \tilde{S}_1 \tilde{d}_1)}{2\delta^2 \lambda \text{Var}(\tilde{C}_1)}. \quad (17)$$

If, in addition, we assume price and demand to be independent, then

$$\text{Cov}(\tilde{C}_1, \tilde{S}_1 \tilde{d}_1) = E(\tilde{d}_1) \text{Cov}(\tilde{S}_1, \tilde{C}_1), \quad (18)$$

and (17) reduces to

$$U_0^* = \frac{\delta E(\tilde{C}_1) - (C_0 + h_c)}{2\lambda \delta^2 \text{Var}(\tilde{C}_1)} + E(\tilde{d}_1) \frac{\text{Cov}(\tilde{S}_1, \tilde{C}_1)}{\text{Var}(\tilde{C}_1)}. \quad (19)$$

Given that the expected discounted price  $\delta E(\tilde{C}_1)$  exceeds the current price-plus-option carrying charges, Equation 19 suggests that the amount of options purchased decreases with price uncertainty and risk aversion and increases with the expected demand.

#### Case 3: Options Available—Risk Neutrality

For  $\lambda = 0$ , the objective function is linear in the decision variables  $V_0$  and  $U_0$ . Specifically,

$$\begin{aligned} E\mathcal{L}(F(V_0, U_0)) &= (S_0 + h_s - \delta E(\tilde{S}_1))V_0 \\ &\quad + (C_0 + h_c - \delta E(\tilde{C}_1))U_0 \\ &\quad + \delta(E(\tilde{S}_1)E(\tilde{d}_1) + \text{Cov}(\tilde{S}_1, \tilde{d}_1)). \end{aligned} \quad (20)$$

In this case, the least-cost solution is to satisfy demand either by initially purchasing options or by inventory building, but not by both. Specifically, from Equation 20 we can see that options will be purchased in lieu of inventory if the expected net profit from options exceeds the expected net profit of inventory. That is,

$$\delta E(\tilde{C}_1) - (C_0 + h_c) > \delta E(\tilde{S}_1) - (S_0 + h_s). \quad (21)$$

#### Case 4: Options Available—Quadratic Utility, Price-Demand Independence

Under the assumption of price-demand uncertainty independence, Equations 8 and 9 reduce to

$$V_0^* = E(\tilde{d}_1) + \frac{b_2 \text{Cov}(\tilde{S}_1, \tilde{C}_1) - b_1 \text{Var}(\tilde{C}_1)}{\text{Cov}^2(\tilde{S}_1, \tilde{C}_1) - \text{Var}(\tilde{S}_1) \text{Var}(\tilde{C}_1)} \quad (22)$$

and

$$U_0^* = \frac{b_1 \text{Cov}(\tilde{S}_1, \tilde{C}_1) - b_2 \text{Var}(\tilde{S}_1)}{\text{Cov}^2(\tilde{S}_1, \tilde{C}_1) - \text{Var}(\tilde{S}_1) \text{Var}(\tilde{C}_1)}. \quad (23)$$

These two equations can be rewritten as

$$V_0^* = E(\tilde{d}_1) + \frac{1}{(1 - \rho_{\tilde{x}}^2)} \left[ \frac{b_1}{\text{Var}(\tilde{S}_1)} - \frac{b_2}{\text{Cov}(\tilde{S}_1, \tilde{C}_1)} \right] \quad (24)$$

and

$$U_0^* = \frac{1}{(1 - \rho_{\tilde{x}}^2)} \left[ \frac{b_2}{\text{Var}(\tilde{C}_1)} - \frac{b_1 \rho_{\tilde{x}}^2}{\text{Cov}(\tilde{S}_1, \tilde{C}_1)} \right], \quad (25)$$

where

$$\rho_{\tilde{x}}^2 = \frac{\text{Cov}^2(\tilde{S}_1, \tilde{C}_1)}{\text{Var}(\tilde{S}_1) \text{Var}(\tilde{C}_1)}. \quad (26)$$

From Equation 25, a positive position will be taken on the option if

$$\frac{b_2}{\text{Var}(\tilde{C}_1)} > \frac{b_1 \rho_{sc}^2}{\text{Cov}(\tilde{S}_1, \tilde{C}_1)} \quad \text{or} \quad b_2 > \beta b_1, \quad (27)$$

where  $\beta = \text{Cov}(\tilde{S}_1, \tilde{C}_1) / \text{Var}(\tilde{S}_1)$  is the slope of the regression line of option prices against commodity prices in period 1 and is referred to as the finite period beta value.

Substituting for  $b_1$  and  $b_2$  into (27) and rearranging, we obtain the following condition, which ensures that options will be purchased:

$$\delta E(C_1) - (C_0 + h_c) > \beta(\delta E(S_1) - (S_0 + h_s)). \quad (28)$$

Equation 28 indicates that options will be purchased in lieu of inventory, provided the expected net profit per option exceeds the beta value times the expected net profit of inventory.

Since the risk aversion parameter  $\lambda$  cancels, the previous result holds regardless of the degree of aversion to risk. Moreover, since  $\beta$  is less than one, the effect of risk aversion, in this case, is to increase the likelihood of incorporating options (compare Equations 21 with 28).

#### Case 5: Purchasing and Options Decisions under Bivariate Lognormality of Prices and Demand

Possibly the most interesting situation occurs when price and demand are correlated and the decision maker is risk averse. In this situation, the general equations (5) and (6) will take on a special form, depending on the statistical distribution imposed on price and demand. In this section, we develop the relevant formulae for these equations under the assumption that price and demand in period 1 have a joint lognormal distribution. This distribution has the desirable feature of avoiding negative outcomes. Moreover, the lognormal distribution has become the prototypical distribution for modeling price behavior in efficient markets (Smith). Let

$$S_1 = S_0 e^{\mu_s + \sigma_s^2 z_s^2/2}, \quad \text{and} \quad (29)$$

$$d_1 = d_0 e^{\mu_d + \sigma_d^2 z_d^2/2}, \quad (30)$$

where  $\mu_s$  and  $\mu_d$  are the logarithmic mean returns, and  $\sigma_s^2$  and  $\sigma_d^2$  are the logarithmic variances for prices and demand, respectively.  $z_s$  and  $z_d$  are standard normal random variables, with

$$\text{Cov}(z_s, z_d) = \rho. \quad (31)$$

Because of the properties of the lognormal distri-

bution, the expectations and variances are given by

$$E(\tilde{S}_1) = S_0 e^{\mu_s + \sigma_s^2/2}, \quad (32)$$

$$E(\tilde{d}_1) = d_0 e^{\mu_d + \sigma_d^2/2},$$

$$\text{Var}(\tilde{S}_1) = E(\tilde{S}_1)^2 (e^{\sigma_s^2} - 1), \quad (33)$$

$$\text{Var}(\tilde{d}_1) = E(\tilde{d}_1)^2 (e^{\sigma_d^2} - 1), \quad \text{and}$$

$$\text{Cov}(\tilde{S}_1, \tilde{d}_1) = E(\tilde{S}_1)E(\tilde{d}_1)(e^{\rho\sigma_s\sigma_d} - 1). \quad (34)$$

In the Appendix, we develop the necessary formulae for computing the other relevant variance and covariance terms. These results are then used in a numerical example to examine conditions under which options can be useful for managing inventory.

#### Example

The data in Table I illustrate the optimal solutions for inventory building and option contracting under a range of values of correlation between the price and the demand. Notice that, in the example, the value incorporating options in purchasing decisions increases as the correlation between price and demand increases.

The last column in Table I indicates the percentage improvement in expected utility that can be obtained by allowing decision makers to incorporate options. Notice that allowing the purchasing agent to utilize option contracts has positive benefits when correlations between price and demand are high. The results in the table suggest that:

- (i) If price and demand are perfectly negatively correlated, the optimal strategy is to defer decisions until resolution of uncertainty occurs.
- (ii) If price and demand are perfectly positively correlated, then purchasing options is an optimal strategy. If demand (and, hence, price) is low, losses from not exercising the options will be less than the potential losses from building inventory.
- (iii) For zero and small negative correlation situations, the optimal strategy in this example is to build inventory in lieu of option purchasing. However, as the correlation increases, the role of options becomes increasingly more important.

#### 4. Conclusion

The options-related literature has emphasized the valuation of options and has paid little attention to their use in building decision making flexibility (and contingency planning) in operations management applications. The purpose of this paper has been to bridge the gap between valuation and decision making by



**Table I**  
Sensitivity of Decisions with Respect to Correlation  
Between Price and Demand

Correlation $\rho$	Inventory Options		Inventory with No Options	Percent Change in Expected Utility
	$V_0$	$U_0$		
-1.0	0.0	0.0	0.0	0.0
-0.8	0.0	0.0	0.0	0.0
-0.6	0.0	0.0	0.0	0.0
-0.4	7.6	0.0	7.6	0.0
-0.2	16.0	0.0	16.0	0.0
0.0	25.0	0.0	25.0	0.0
0.2	26.5	10.1	34.9	0.4
0.4	25.1	24.8	45.7	1.3
0.6	20.2	44.9	57.4	2.7
0.8	11.7	70.4	70.1	4.5
1.0	0.0	101.2	83.8	6.5

Case Parameters:  $S_0 = 20$ ,  $d_0 = 20$ ,  $U_s = 0.15$ ,  $U_d = 0.10$ ,  $\sigma_s = 0.3$ ,  $\sigma_d = 0.5$ ,  $h_s = 0.10S_0$ ,  $h_d = 0.09C_0$ ,  $\pi = 20$ ,  $\delta = 0.9$ ,  $r = 0.1$ ,  $\lambda = 0.5$ .

applying options as a risk management instrument for hedging against price and quantity uncertainty in inventory procurement decisions.

In a setting of risk neutrality, the choice between options and inventory building is determined by a simple ratio test (Equation 21) involving the net expected profit associated with carrying options relative to carrying stock. With quadratic risk aversion, we have developed conditions for an optimal portfolio of options and inventory building. In the event that price and demand uncertainties are uncorrelated, simplifications occur, and we have shown that options will be purchased in lieu of inventory, provided the ratio of expected net profit per option to the expected net profit in inventory exceeds a beta value which is less than one (Equation 27). In this case, since the risk aversion parameter cancels, the choice between options and stock building will not vary among managers with different risk averse quadratic utility preferences. Section 3 provided a more general example, with price and demand modeled as joint lognormal random variables. In such situations the use of options depends on price-demand correlations. Our examples illustrate that if price and demand are strongly negatively correlated, then neither options nor inventory building are appropriate. In this case, delaying decisions is more suitable. When demand and price are weakly or strongly positively correlated, option contracts become more desirable. Finally, the benefits of contingent claims contracting with options allow regret decision making and provide a hedge against unforeseen and significant changes in demands and prices. This article has illustrated how such contingent claims can mesh with purchasing orders/inventory building in an effective risk-management controlled program.

### Appendix

Assume that the future spot price  $S_1$  and demand  $d_1$  follow a joint lognormal distribution.

Then

$$S_1 = S_0 e^{\mu_s + \sigma_s \tilde{z}_s} \quad \text{and} \quad d_1 = d_0 e^{\mu_d + \sigma_d \tilde{z}_d},$$

where  $\tilde{z}_s$  and  $\tilde{z}_d$  are joint standard normal random variables with  $E(\tilde{z}_s \tilde{z}_d) = \rho$ . Let  $C_1$  be the future price of a call option with strike price,  $\pi$ . The Black-Scholes equation for the price of a call option is

$$C_0 = e^{-rT} \hat{C}(r),$$

where

$$\hat{C}(r) = S_0 N(d(r)) - \pi N(d(r) - \sigma)$$

and

$$d(r) = \left( \ln \left( \frac{S_0}{\pi} \right) + r + \frac{\sigma^2}{2} \right) / \sigma,$$

and  $N(\cdot)$  is the cumulative normal distribution function.

The next few lemmas develop the expressions necessary for computing the optimal solutions for Equations 7 and 8.

#### Lemma 1.

$$\text{Cov}(\tilde{S}_1, \tilde{d}_1) = E(\tilde{S}_1)E(\tilde{d}_1)(e^{\rho\sigma_s\sigma_d} - 1).$$

#### Proof

$$\begin{aligned} S_1 d_1 &= S_0 d_0 \exp(\mu_s + \mu_d + \sigma_s \tilde{z}_s + \sigma_d \tilde{z}_d) \\ &= S_0 d_0 \exp(\mu_s + \mu_d + (\sigma_s^2 + \sigma_d^2 + 2\rho\sigma_s\sigma_d)^{1/2} \tilde{z}), \end{aligned}$$

Hence

$$\begin{aligned} E(\tilde{S}_1 \tilde{d}_1) &= S_0 d_0 \exp(\mu_s + \mu_d + (\sigma_s^2 + \sigma_d^2 + 2\rho\sigma_s\sigma_d)^{1/2} \bar{z}), \\ &= E(\tilde{S}_1)E(\tilde{d}_1)e^{\rho\sigma_s\sigma_d}. \end{aligned} \quad (A1)$$

Now

$$\text{Cov}(\tilde{S}_1, \tilde{d}_1) = E(\tilde{S}_1 \tilde{d}_1) - E(\tilde{S}_1)E(\tilde{d}_1). \quad (A2)$$

Substituting (A1) into (A2) and simplifying yields the result.

**Lemma 2.**

$$\text{Cov}(\tilde{S}_1, \tilde{S}_1 \tilde{d}_1) = E(\tilde{S}_1)^2 E(\tilde{d}_1) e^{\rho\sigma_s\sigma_d} [e^{\rho\sigma_s\sigma_d - \sigma_s^2} - 1].$$

**Proof**

$$S_1^2 d_1 = S_0^2 d_0 \exp(2\mu_s + \mu_d + (4\sigma_s^2 + \sigma_d^2 + 4\rho\sigma_s\sigma_d)^{1/2} \bar{z}).$$

Hence

$$E(\tilde{S}_1^2 \tilde{d}_1) = E(\tilde{S}_1)^2 E(\tilde{d}_1) (\exp(2\rho\sigma_s\sigma_d + \sigma_s^2)). \quad (A3)$$

From (A1), we have

$$E(\tilde{S}_1)E(\tilde{S}_1 \tilde{d}_1) = E(\tilde{S}_1)^2 E(\tilde{d}_1) e^{\rho\sigma_s\sigma_d}. \quad (A4)$$

Now,

$$\text{Cov}(\tilde{S}_1, \tilde{S}_1 \tilde{d}_1) = E(\tilde{S}_1^2 \tilde{d}_1) - E(\tilde{S}_1)E(\tilde{S}_1 \tilde{d}_1). \quad (A5)$$

Substituting (A3) and (A4) into (A5) yields the result.

**Lemma 3.**

$$\begin{aligned} E(\tilde{C}_1) &= \hat{C}(m), \\ \text{Cov}(\tilde{S}_1, \tilde{C}_1) &= E(\tilde{S}_1)[\hat{C}(m + \sigma_s^2) - \hat{C}(m)], \\ \text{and} \\ \text{Var}(\tilde{C}_1) &= E(\tilde{S}_1)\hat{C}(m + \sigma_s^2) \\ &\quad - \hat{C}(m)(X + \hat{C}(m)), \end{aligned} \quad (A6)$$

where  $m = \mu_s + \sigma_s^2/2$ .

**Proof**

$$\begin{aligned} P(\tilde{S}_1 > X) &= P(S_0 \exp(\mu_s + \sigma_s \bar{z}_s) > \pi), \\ &= P\left(\bar{z}_s > -\frac{\ln(S_0/\pi) + \mu_s}{\sigma_s}\right), \\ &= N(d(m) - \sigma_s). \end{aligned} \quad (A8)$$

Furthermore, for  $k = 1, 2$  we have

$$\begin{aligned} E(\tilde{S}_1^k | \tilde{S}_1 > \pi)P(\tilde{S}_1 > \pi) &= E(S_0^k e^{k\mu_s + k\sigma_s \bar{z}_s} | \bar{z}_s > -d(m) + \sigma_s)N(d(m) - \sigma_s), \\ &= S_0^k e^{k\mu_s} (2\pi)^{-1/m} \int_{-d(m)+\sigma_s}^{\infty} e^{k\sigma_s \bar{z}_s - \bar{z}_s^2/2} d\bar{z}_s. \end{aligned}$$

Completing the square and simplifying reduces the expression to

$$\begin{aligned} E(\tilde{S}_1^k | \tilde{S}_1 > \pi)P(\tilde{S}_1 > \pi) &= S_0^k e^{k\mu_s + k^2\sigma_s^2/2} N(d(m) + (k-1)\sigma_s) \end{aligned} \quad (A9)$$

Given Equations A8 and A9, we can compute the expectation, covariance, and variance expressions. Specifically,

$$E(\tilde{C}_1) = E(\tilde{S}_1 | \tilde{S}_1 > \pi)P(\tilde{S}_1 > \pi) - XP(\tilde{S}_1 > \pi). \quad (A10)$$

Substituting Equations A9 (with  $k = 1$ ) and A8 into A10 yields A6. Similarly,

$$\text{Cov}(\tilde{S}_1, \tilde{C}_1) = E(\tilde{S}_1 \tilde{C}_1) - E(\tilde{S}_1)E(\tilde{C}_1) \quad (A11)$$

and

$$\begin{aligned} E(\tilde{S}_1 \tilde{C}_1) &= E(\tilde{S}_1^2 | \tilde{S}_1 > \pi)P(\tilde{S}_1 > \pi) \\ &\quad - XE(\tilde{S}_1 | \tilde{S}_1 > \pi)P(\tilde{S}_1 > \pi). \end{aligned} \quad (A12)$$

Substituting (A9) (with  $k = 1$  and 2) into (A12) and simplifying yields

$$E(\tilde{S}_1 \tilde{C}_1) = E(\tilde{S}_1)\hat{C}(m + \sigma_s^2). \quad (A13)$$

Substituting (A13) into (A11) yields (A6).

Finally,

$$\text{Var}(\tilde{C}_1) = E(\tilde{C}_1^2) - E(\tilde{C}_1)^2 \quad (A14)$$

and

$$\begin{aligned} E(\tilde{C}_1^2) &= E(\tilde{S}_1^2 | \tilde{S}_1 > \pi)P(\tilde{S}_1 > \pi) \\ &\quad - 2XE(\tilde{S}_1 | \tilde{S}_1 > \pi)P(\tilde{S}_1 > \pi) \\ &\quad + \pi P(\tilde{S}_1 > \pi), \\ &= E(\tilde{S}_1 \tilde{C}_1) - \pi(E(\tilde{C}_1)), \\ &= E(\tilde{S}_1)\hat{C}(m + \sigma_s^2) - \pi\hat{C}(m). \end{aligned} \quad (A15)$$

Substituting (A15) into (A14) yields (A7).

**Lemma 4.**

$$\begin{aligned} \text{Cov}(\tilde{C}_1, \tilde{S}_1 \tilde{d}_1) &= E(\tilde{S}_1)E(\tilde{d}_1)e^{\rho\sigma_s\sigma_d} [e^{\rho\sigma_s\sigma_d + \sigma_s^2} E(\tilde{S}_1)N(b_1) - \pi N(b_2)] \\ &\quad - E(\tilde{C}_1), \end{aligned} \quad (A16)$$

where

$$b_1 = d(m) + \rho\sigma_d,$$

$$b_2 = b_1 + \sigma_s.$$

## Proof

$$\begin{aligned}
 & E(\tilde{S}_1^* \tilde{d}_1 | S_1 > \pi) P(\tilde{S}_1 > \pi) \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_0^k d_0 e^{k\mu_s + \mu_d + k\sigma_s z_s + \sigma_d z_d} \\
 &\quad \cdot f(z_s, z_d) dz_s dz_d, \quad (A17)
 \end{aligned}$$

where  $a' = -d(m) + \sigma_s$  and  $f(z_s, z_d)$  is the bivariate normal density of  $z_s$  and  $z_d$ .

Let  $U = k\sigma_s z_s$ ,  $V = \sigma_d z_d$ . Then (A17) is given by

$$\begin{aligned}
 & E(\tilde{S}_1^* \tilde{d}_1 | \tilde{S}_1 > \pi) \\
 &= S_0^k d_0 e^{k\mu_s + \mu_d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{u + av} f(u, v) du, dv \quad (A18)
 \end{aligned}$$

where  $a = a' k \sigma_s$ .

Rubinstein (1976) shows that the integral in (A18) equals

$$e^{1/2(\sigma_s^2 + \sigma_d^2 + 2\rho\sigma_s\sigma_d)} N\left(-\frac{a}{\sigma_u} + \sigma_u + \rho\sigma_v\right).$$

Hence (A18) is given by

$$\begin{aligned}
 & E(\tilde{S}_1^* \tilde{d}_1 | \tilde{S}_1 > \pi) P(\tilde{S}_1 > \pi) \\
 &= S_0^k d_0 e^{k\mu_s + \mu_d} e^{1/2(k^2\sigma_s^2 + \sigma_d^2 + 2k\rho\sigma_s\sigma_d)} \\
 &\quad \cdot N(d(m) + (k-1)\sigma_s^2 + \rho\sigma_d). \quad (A19)
 \end{aligned}$$

Now

$$\begin{aligned}
 E(\tilde{C}_1 \tilde{S}_1 \tilde{d}_1) &= E(\tilde{S}_1^* \tilde{d}_1 | \tilde{S}_1 > \pi) P(\tilde{S}_1 > \pi) \\
 &\quad - X E(\tilde{S}_1 \tilde{d}_1 | \tilde{S}_1 > \pi) P(\tilde{S}_1 > \pi) \quad (A20)
 \end{aligned}$$

Substituting (A19) (for  $k = 1$  and 2) into (A20), and using the result

$$\text{Cov}(\tilde{C}_1, \tilde{S}_1 \tilde{d}_1) = E(\tilde{C}_1 \tilde{S}_1 \tilde{d}_1) - E(\tilde{C}_1) E(\tilde{S}_1 \tilde{d}_1) \quad (A21)$$

yields the results.

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